La science quantique

Une vision singulière

XII) Coupleurs

P.A. Besse

Hamiltonien de couplage

Etudions les Hamiltoniens de la forme:

$$H = \begin{pmatrix} E_0 & -T \\ -T & E_1 \end{pmatrix}$$

PAB

Rappel:
Hamiltoniens et propagateurs

Hamiltonien général pour un single Qubit: (Matrice hermitienne)

$$H = \hbar\omega_m \cdot \mathbb{1} + \frac{\hbar\Omega}{2} \cdot \left(n_x\sigma_x + n_y\sigma_y + n_z\sigma_z \right)$$

Energie commune

Propagateur général pour un single Qubit:

$$U(t) = e^{-i\frac{H}{\hbar}t} = e^{-i\cdot\omega_m\cdot t} \cdot \left(\cos\left(\frac{\Omega}{2}t\right) \cdot \mathbb{1} - i\sin\left(\frac{\Omega}{2}t\right) \cdot \left(n_x\sigma_x + n_y\sigma_y + n_z\sigma_z \right) \right)$$

Phase commune

Rotation d'angle Ω.t de la sphère de Bloch autour de l'axe $\vec{n} = \begin{pmatrix} n_x & n_y & n_z \end{pmatrix}$

1) Fréquence de Larmor

(T=0)

Fréquence de Larmor

Hamiltonien:

$$H = \begin{pmatrix} E_0 & 0 \\ 0 & E_1 \end{pmatrix} = \bar{E} \cdot 1 - \frac{\Delta E}{2} \cdot \sigma_z$$

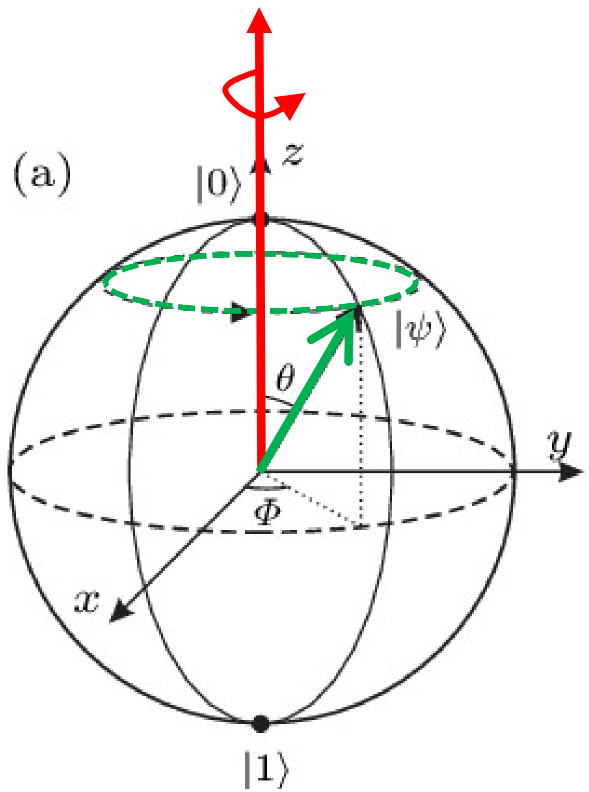
$$\bar{E} \equiv \frac{E_0 + E_1}{2} \qquad \Delta E \equiv E_1 - E_0$$

Propagateur:

$$U(t) = e^{-i\frac{H}{\hbar}t} = e^{-i\cdot\frac{\bar{E}}{\hbar}t} \cdot \left(\cos\left(\frac{1}{2}\frac{\Delta E}{\hbar}t\right) \cdot 1 + i \sin\left(\frac{1}{2}\frac{\Delta E}{\hbar}t\right) \cdot \sigma_z \right) = e^{-i\frac{\bar{E}}{\hbar}t} \cdot \begin{pmatrix} e^{+i\frac{1}{2}\frac{\Delta E}{\hbar}t} & 0 \\ 0 & e^{-i\frac{1}{2}\frac{\Delta E}{\hbar}t} \end{pmatrix}$$

Fréquence de Larmor

$$\Omega_L \equiv \frac{\Delta E}{\hbar}$$

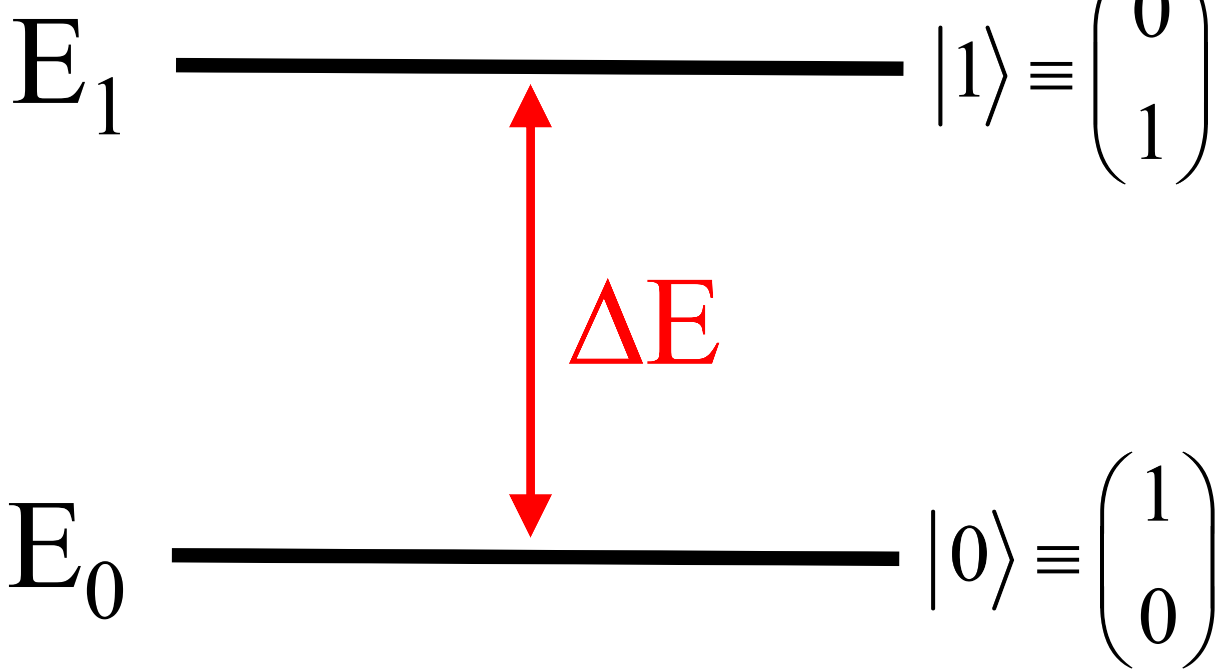


→ Rotation autour de l'axe Z

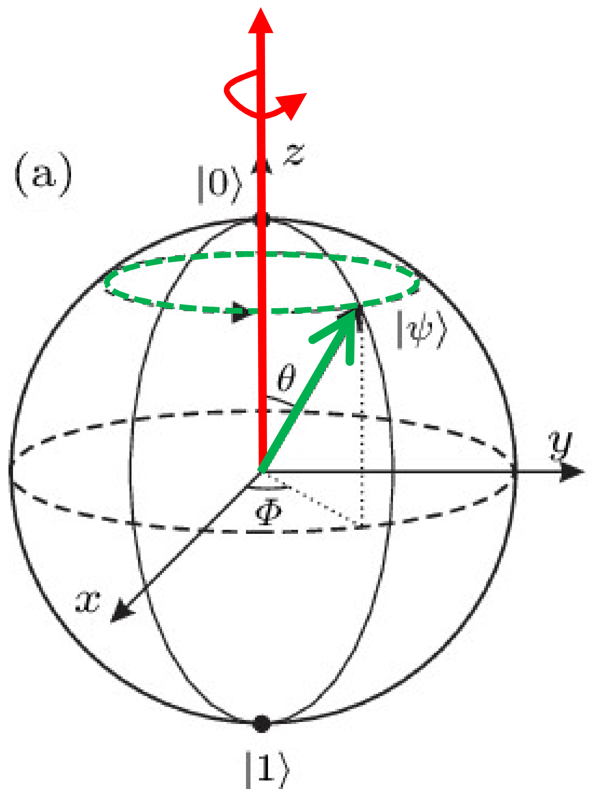
Niveaux d'énergie

Rotation autour
de l'axe Z

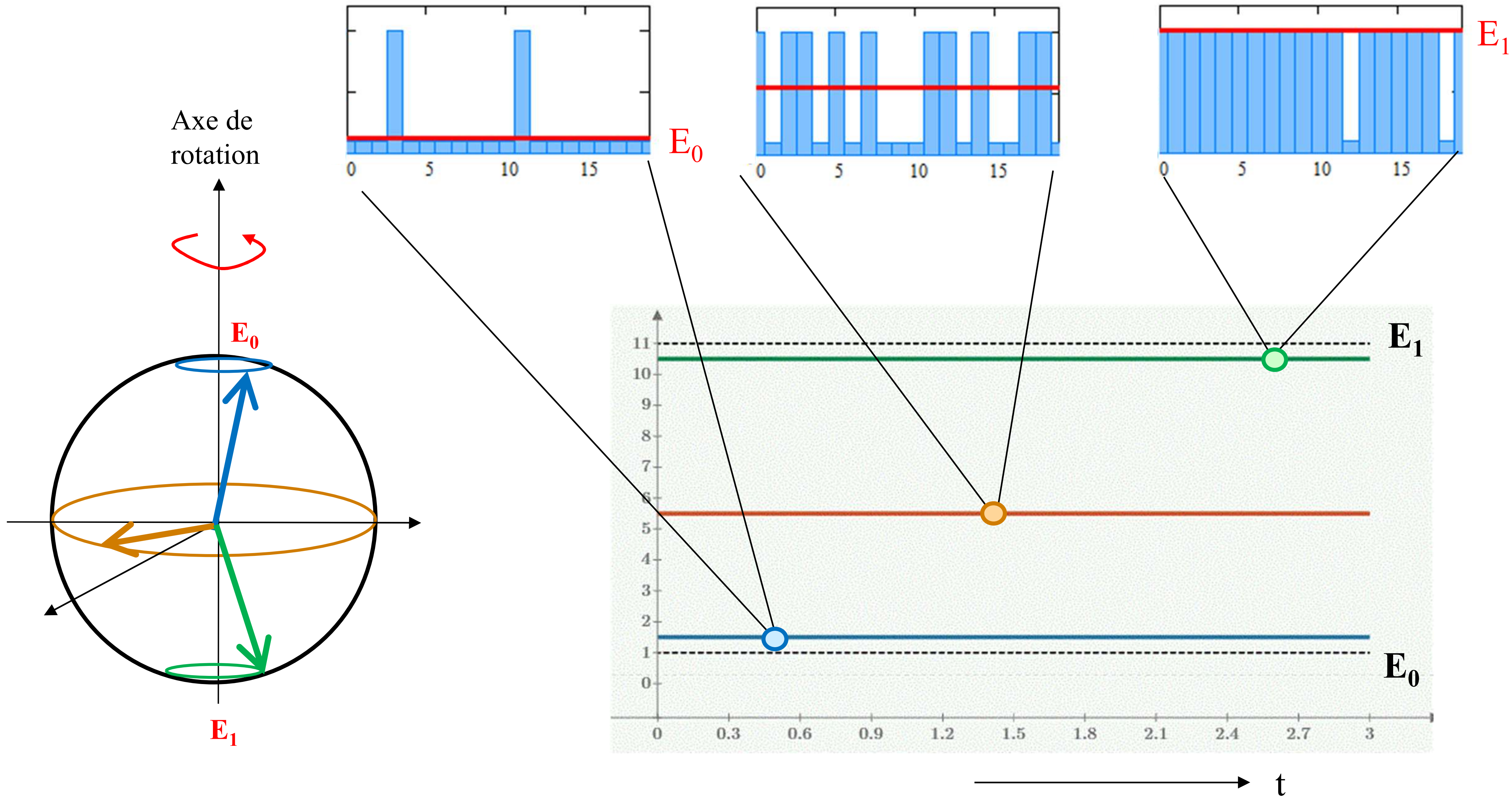
$$\Delta E \equiv \hbar \cdot \Omega_L$$



Axe de rotation



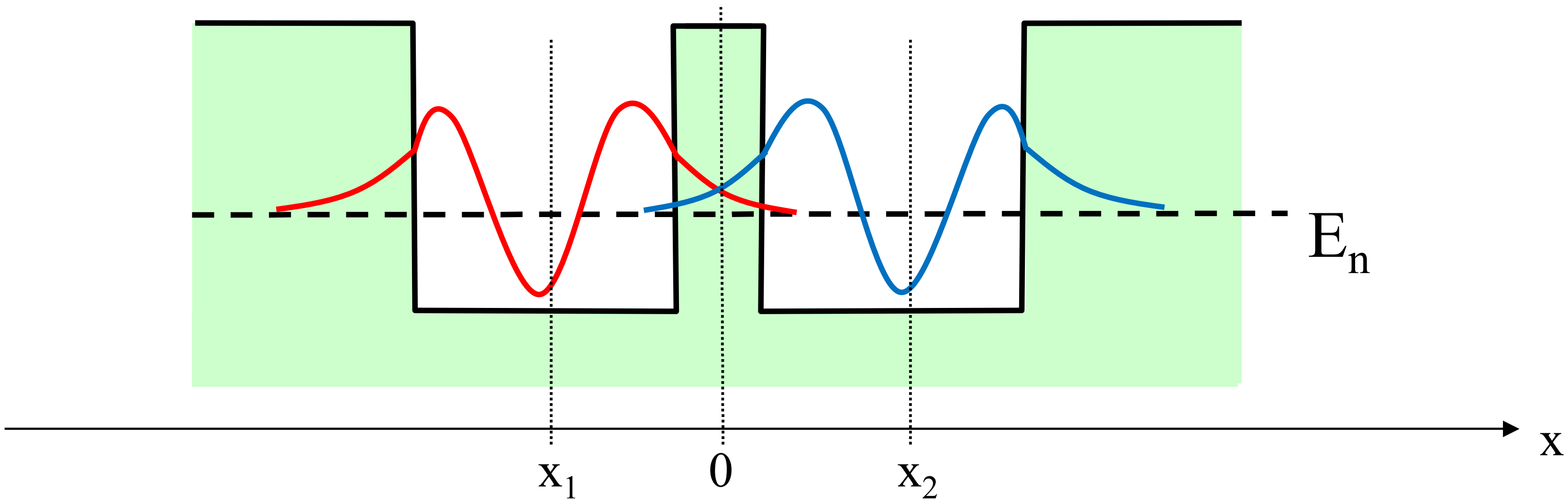
Rappel: mesures singulières et moyenne



2) Double puits symétriques couplés

(E_0=E_1)

Deux puits couplés



Théorie des modes couplés:

$$i\hbar \cdot \frac{\partial}{\partial t}\begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix} = \begin{pmatrix}\bar{E} & -T \\ -T & \bar{E}\end{pmatrix} \cdot \begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix}$$

Double puits symétriques couplés

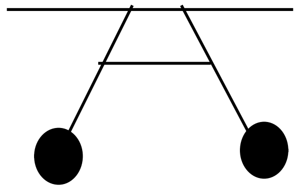
Hamiltonien:

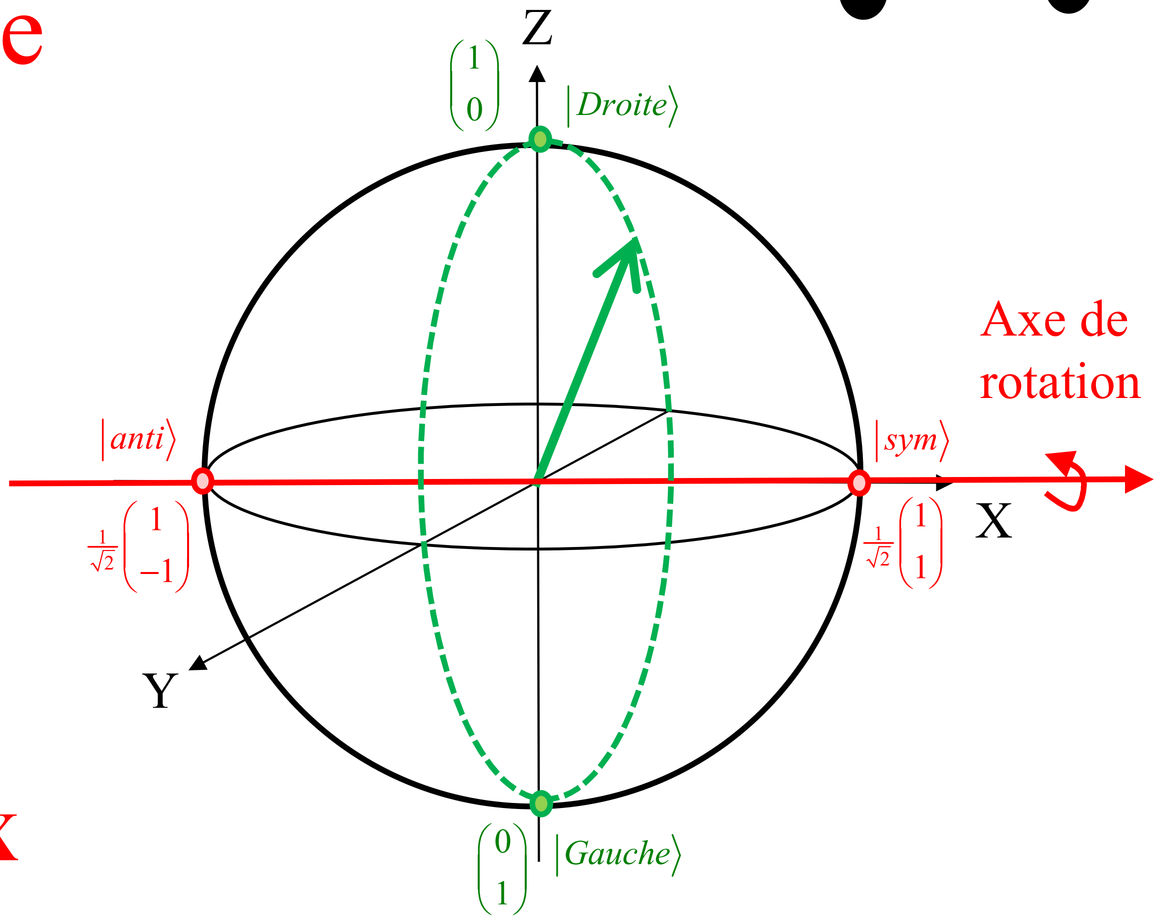
$$H = \begin{pmatrix} \bar{E} & -T \\ -T & \bar{E} \end{pmatrix} = \bar{E} \cdot 1 - T \cdot \sigma_X$$

Propagateur:

$$U(t) = e^{-i\frac{H}{\hbar}t} = e^{-i\cdot\frac{\bar{E}}{\hbar}t} \cdot \left(\cos\left(\frac{1}{2}\frac{2T}{\hbar}t\right) \cdot 1 + i\,\sin\left(\frac{1}{2}\frac{2T}{\hbar}t\right) \cdot \sigma_X \right) = e^{-i\frac{\bar{E}}{\hbar}t} \cdot \begin{pmatrix} \cos\left(\frac{1}{2}\frac{2T}{\hbar}t\right) & i\,\sin\left(\frac{1}{2}\frac{2T}{\hbar}t\right) \\ i\,\sin\left(\frac{1}{2}\frac{2T}{\hbar}t\right) & \cos\left(\frac{1}{2}\frac{2T}{\hbar}t\right) \end{pmatrix}$$

Fréquence de couplage



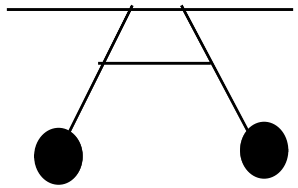


$$\Omega_C \equiv \frac{2T}{\hbar}$$

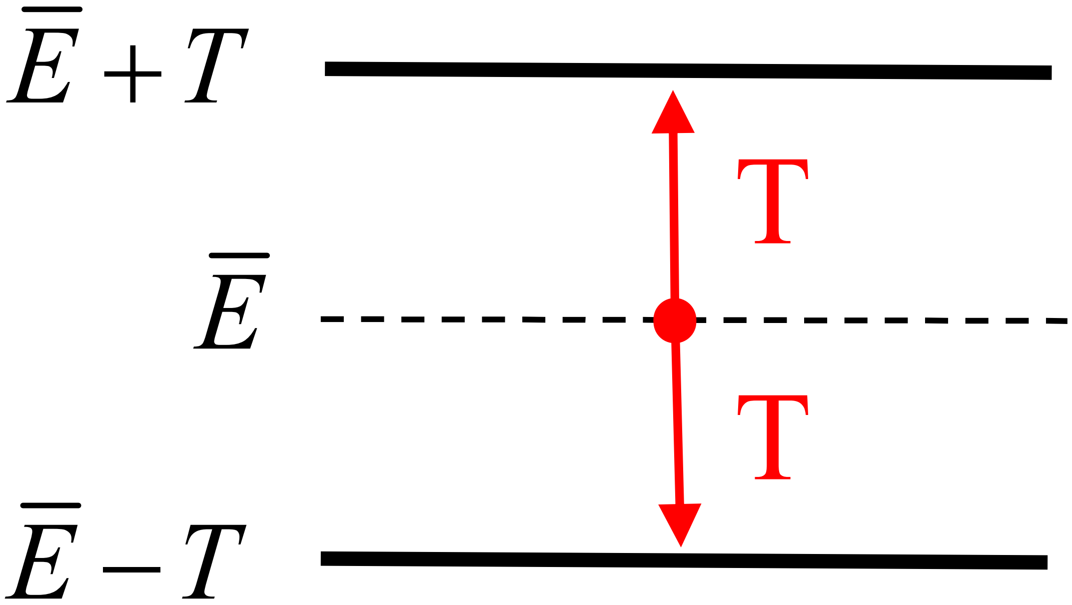
→ Rotation autour de l'axe X

Niveaux d’énergie

Rotation autour
de l’axe X



$$2T \equiv \hbar \cdot \Omega_C$$



$$|anti\rangle \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

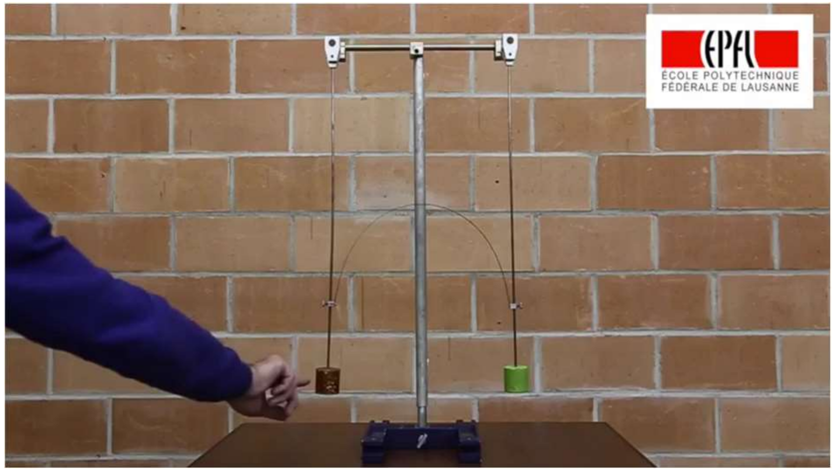
$$|sym\rangle \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

Pendules couplés: battements

https://www.youtube.com/watch?v=aFacOh9hW9U

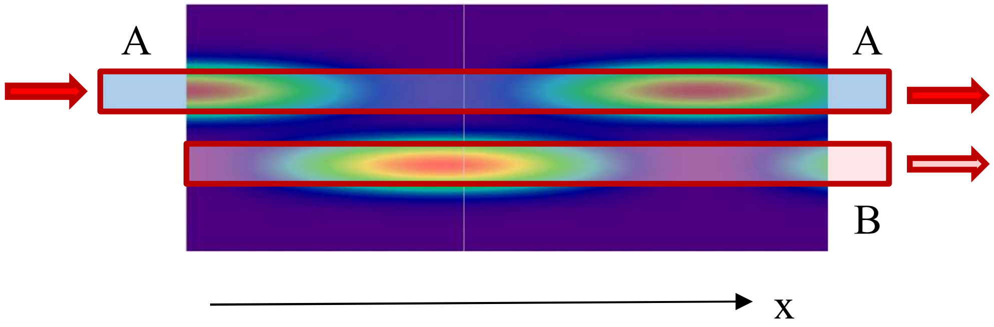
Probabilité «Gauche»:

$$P_G(t) = \left| \left\langle (1,0) \middle| \psi^{out} \right\rangle \right|^2 = \cos^2\left(\frac{1}{2} \frac{2T}{\hbar} \tau \right)$$



Probabilité «Droite»:

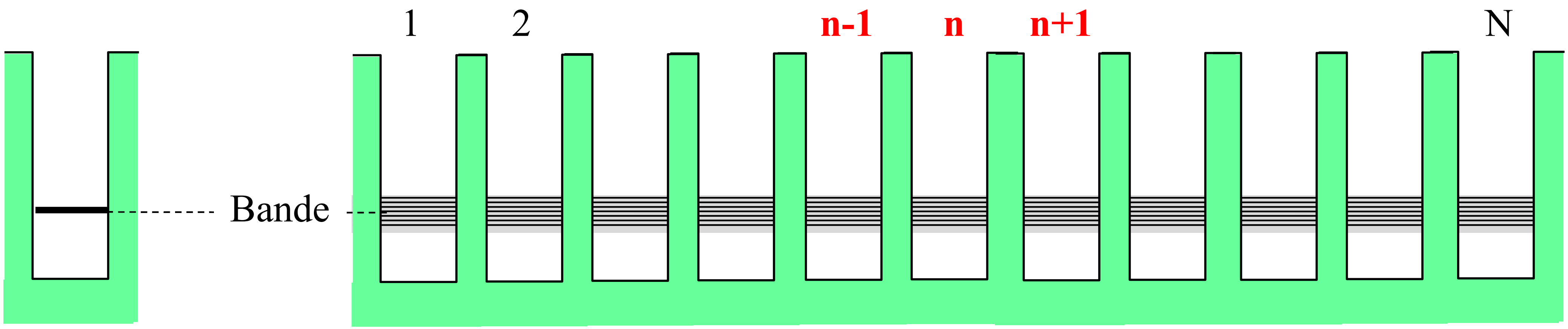
$$P_D(t) = \left| \left\langle (0,1) \middle| \psi^{out} \right\rangle \right|^2 = \sin^2\left(\frac{1}{2} \frac{2T}{\hbar} \tau \right)$$



Formation de bandes

PAB

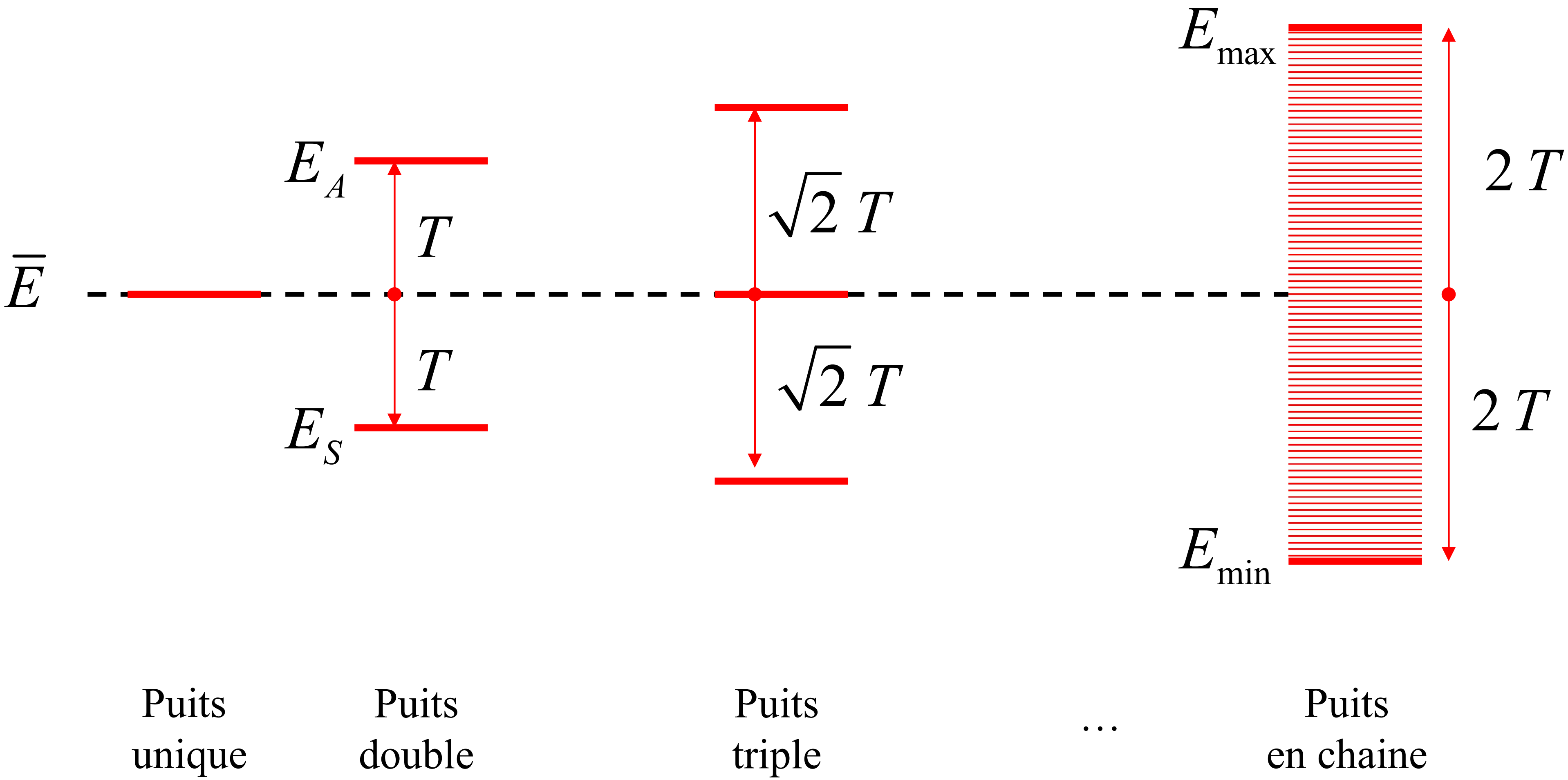
Rappel:
Chaine de puits couplés



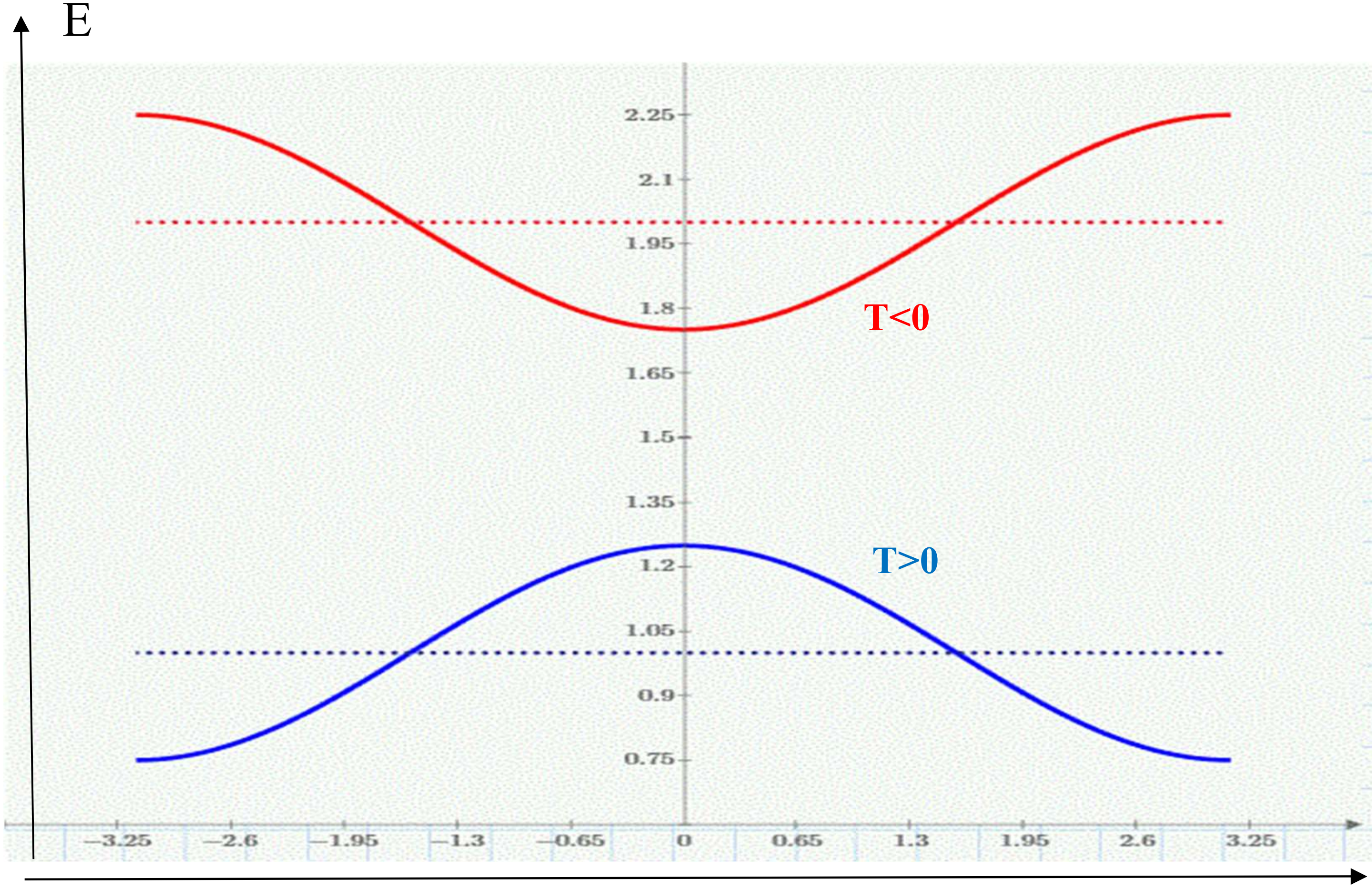
$$H = \begin{pmatrix} \bar{E} & -T & 0 & 0 & 0 & 0 & 0 & 0 \\ -T & \bar{E} & -T & 0 & 0 & 0 & 0 & 0 \\ 0 & \dots & \dots & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & -T & \bar{E} & -T & 0 & 0 & 0 \\ 0 & 0 & 0 & -T & \bar{E} & -T & 0 & 0 \\ 0 & 0 & 0 & 0 & -T & \bar{E} & -T & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & \dots & -T \\ 0 & 0 & 0 & 0 & 0 & 0 & -T & \bar{E} \end{pmatrix}$$

$$|\psi\rangle \cong \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{n-1} \\ \alpha_n \\ \alpha_{n+1} \\ \vdots \\ \alpha_N \end{pmatrix}$$

Chaine de puits couplés



Relation de dispersion et bandes



$$E(\delta) = \bar{E} - 2T \cdot \cos(\delta)$$

$$\delta = n \cdot \frac{2\pi}{N} \qquad n = -\frac{N}{2}, \ldots, \frac{N-1}{2}$$

3) Référentiel tournant et couplage AC

Couplage AC et référentiel tournant

Référentiel fixe:

$$H = \begin{pmatrix} E_0 & -i \cdot T \cdot \sin(\omega t) \\ i \cdot T \cdot \sin(\omega t) & E_1 \end{pmatrix} = \bar{E} \cdot 1 - \frac{\Delta E}{2} \cdot \sigma_z + T \sin(\omega t) \cdot \sigma_y$$

Référentiel tournant à fréquence ω:

$$\bar{H} = \begin{pmatrix} E_0 & -T/2 \\ -T/2 & E_1 - \hbar\omega \end{pmatrix} = \left(\bar{E} - \frac{\hbar\omega}{2} \right) \cdot 1 - \frac{\left(\hbar\Omega_L - \hbar\omega\right)}{2} \cdot \sigma_Z - \frac{T}{2} \cdot \sigma_X$$

Avec la fréquence de Larmor: $\Omega_L = \Delta E / \hbar$

Fréquence de Rabi

Dans le référentiel tournant

$$\bar{H} = \left(\bar{E} - \frac{\hbar\omega}{2} \right) \cdot 1 - \frac{\hbar\Omega_R}{2} \cdot \left(\bar{n}_x \sigma_x + \bar{n}_z \sigma_z \right)$$

Fréquence de Rabi:

$$\hbar\Omega_R \equiv \sqrt{\hbar^2 \left(\Omega_L - \omega \right)^2 + T^2}$$

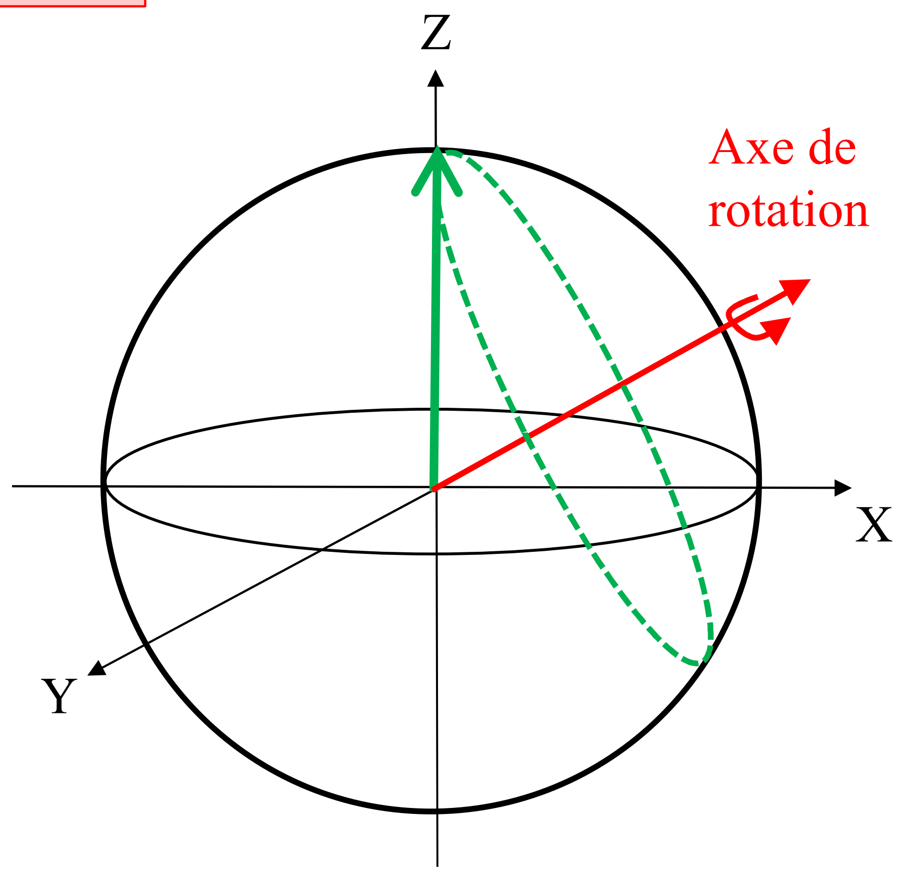
Axe de rotation:

Dans le référentiel tournant

$$\bar{n}_x = \frac{T}{\hbar\Omega_R}$$

$$\bar{n}_y = 0$$

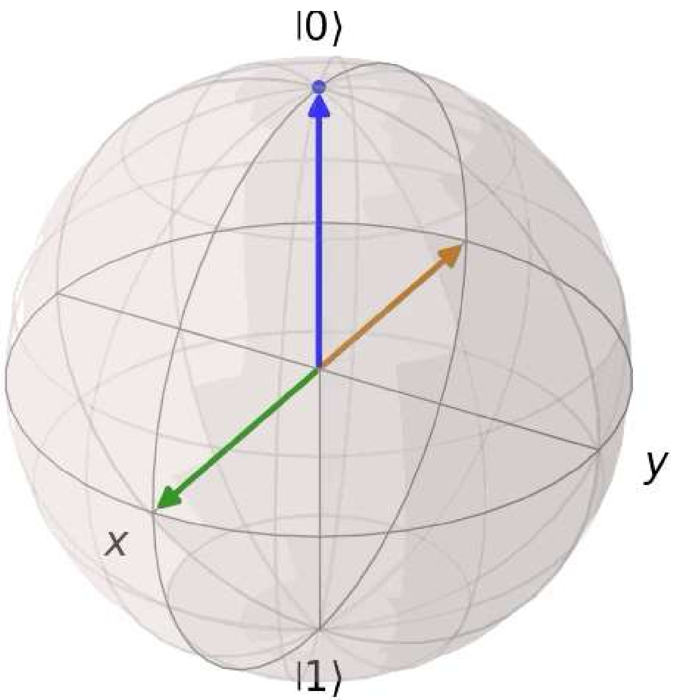
$$\bar{n}_z = \frac{\Omega_L - \omega}{\Omega_R}$$



Rappel Fréquence de Rabi: Effet du detuning

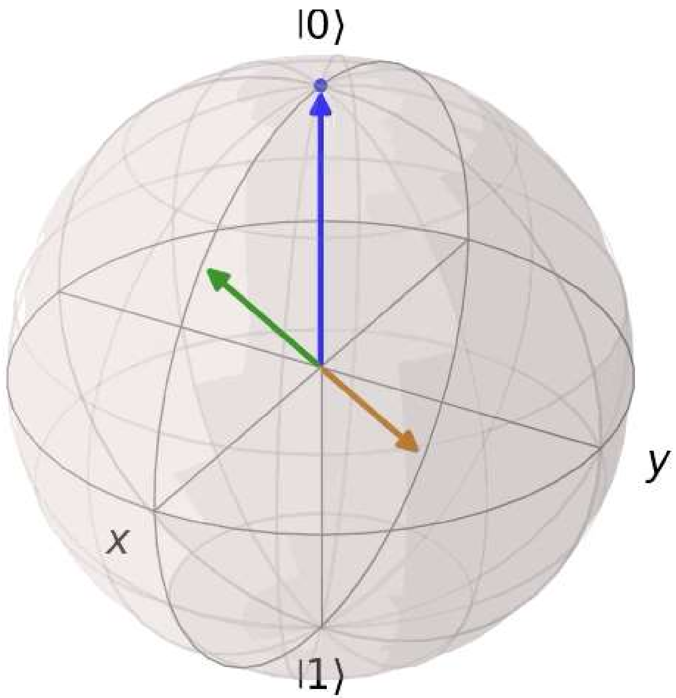
$\omega = \Omega_L$

Résonnant



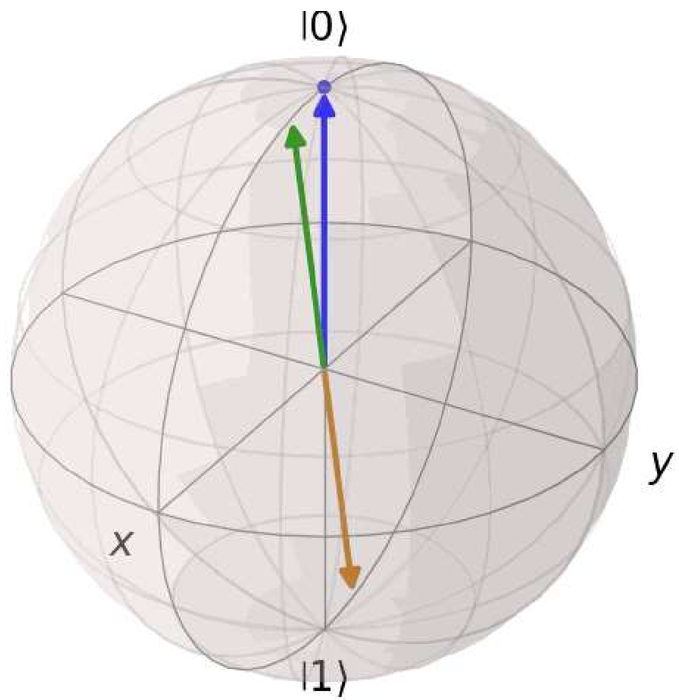
$\omega \neq \Omega_L$

Detuned



$\omega \neq\neq \Omega_L$

Strongly detuned



Simulations par: Romain Nicolas Paul Couyoumtzelis

Fréquence de Larmor et de Rabi couplage résonnant

$$H = \begin{pmatrix} E_0 & -T\cos(\omega t) \\ -T\cdot\cos(\omega t) & E_1 \end{pmatrix}$$

Larmor

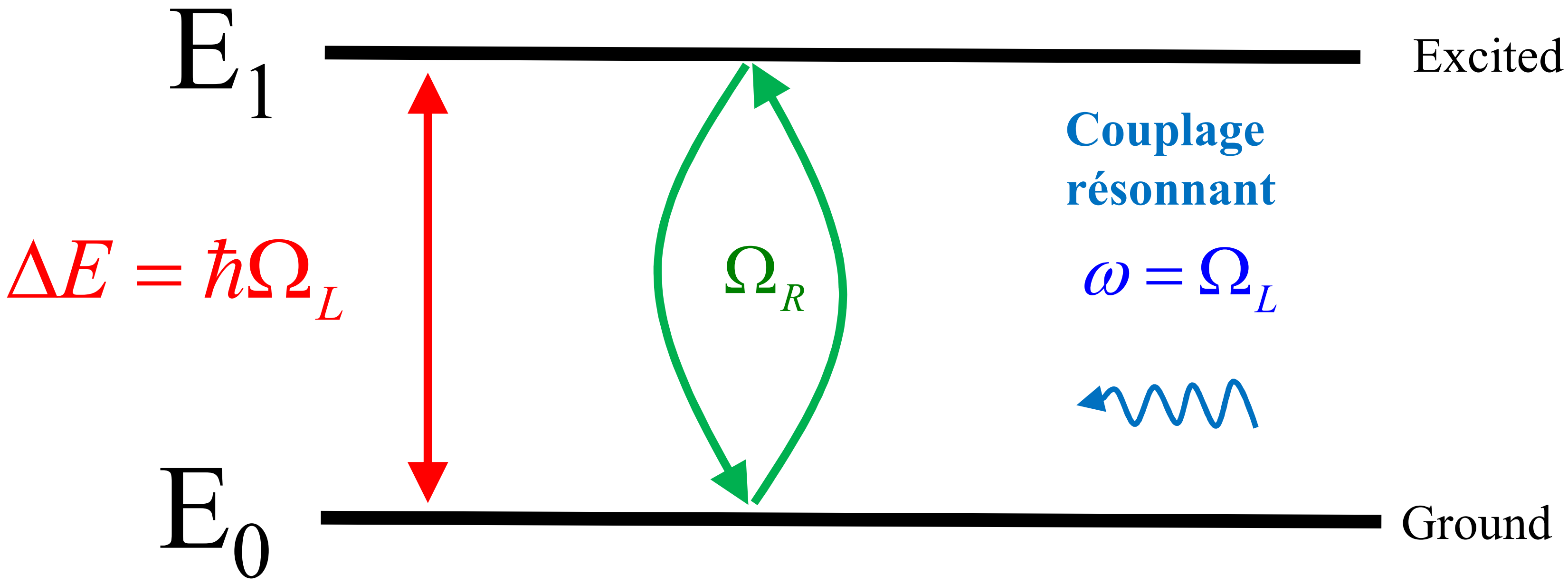
$$\Omega_L \equiv \frac{E_1 - E_0}{\hbar}$$

Résonnance

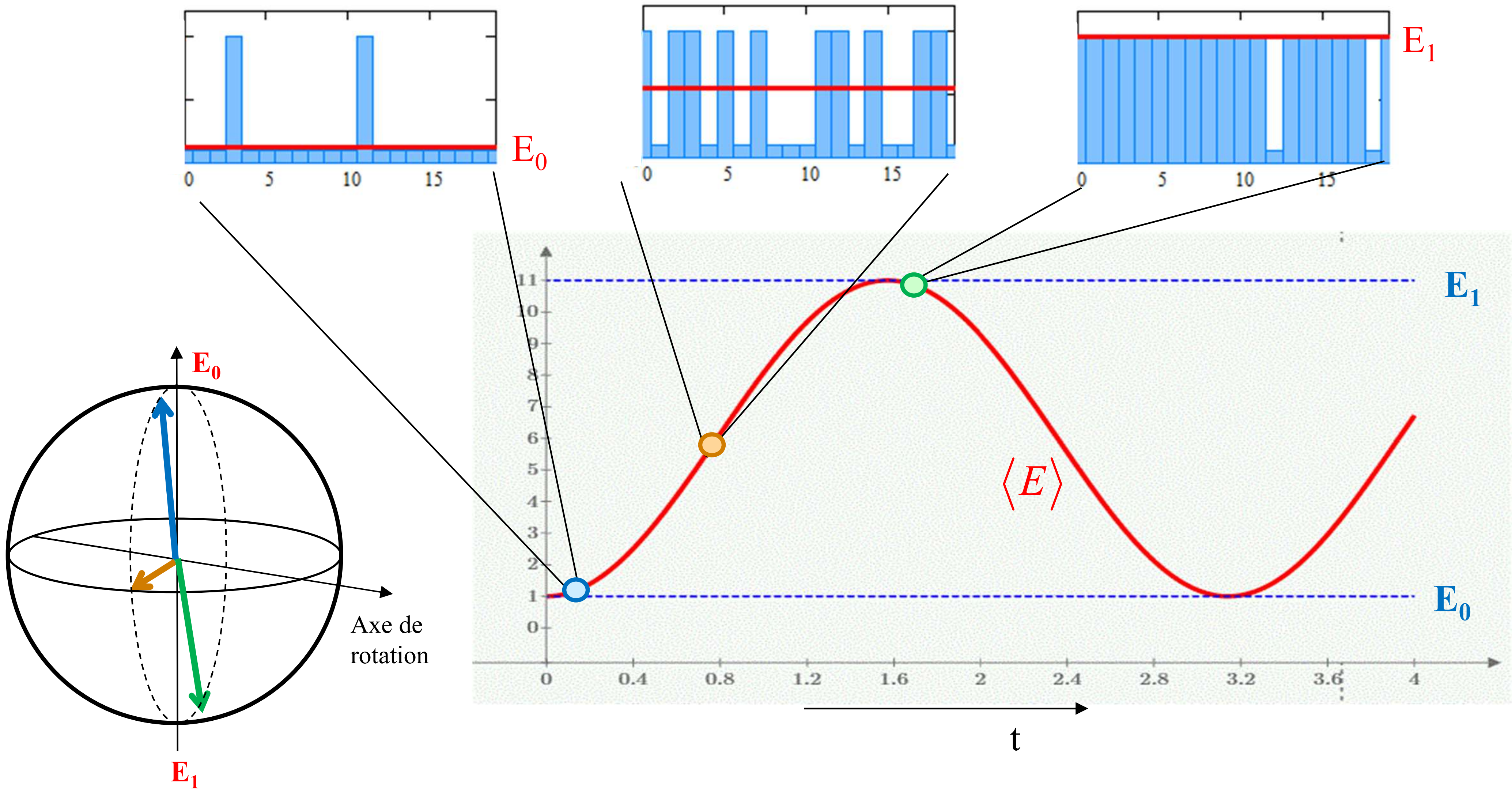
$$\omega = \Omega_L$$

Rabi

$$\Omega_R \equiv \frac{T}{\hbar}$$



Rappel: mesures singulières et moyenne

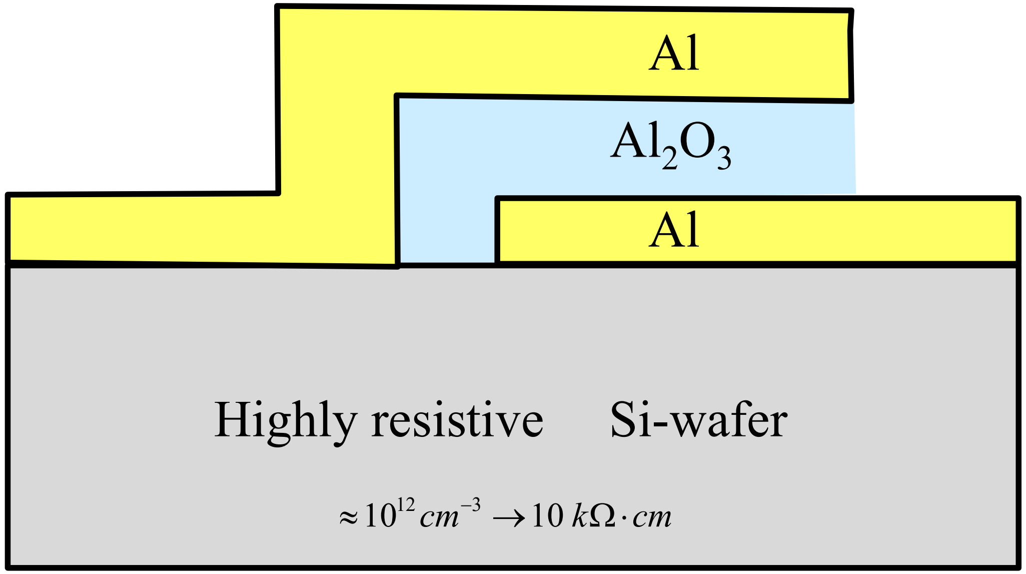


Jonction de Josephson

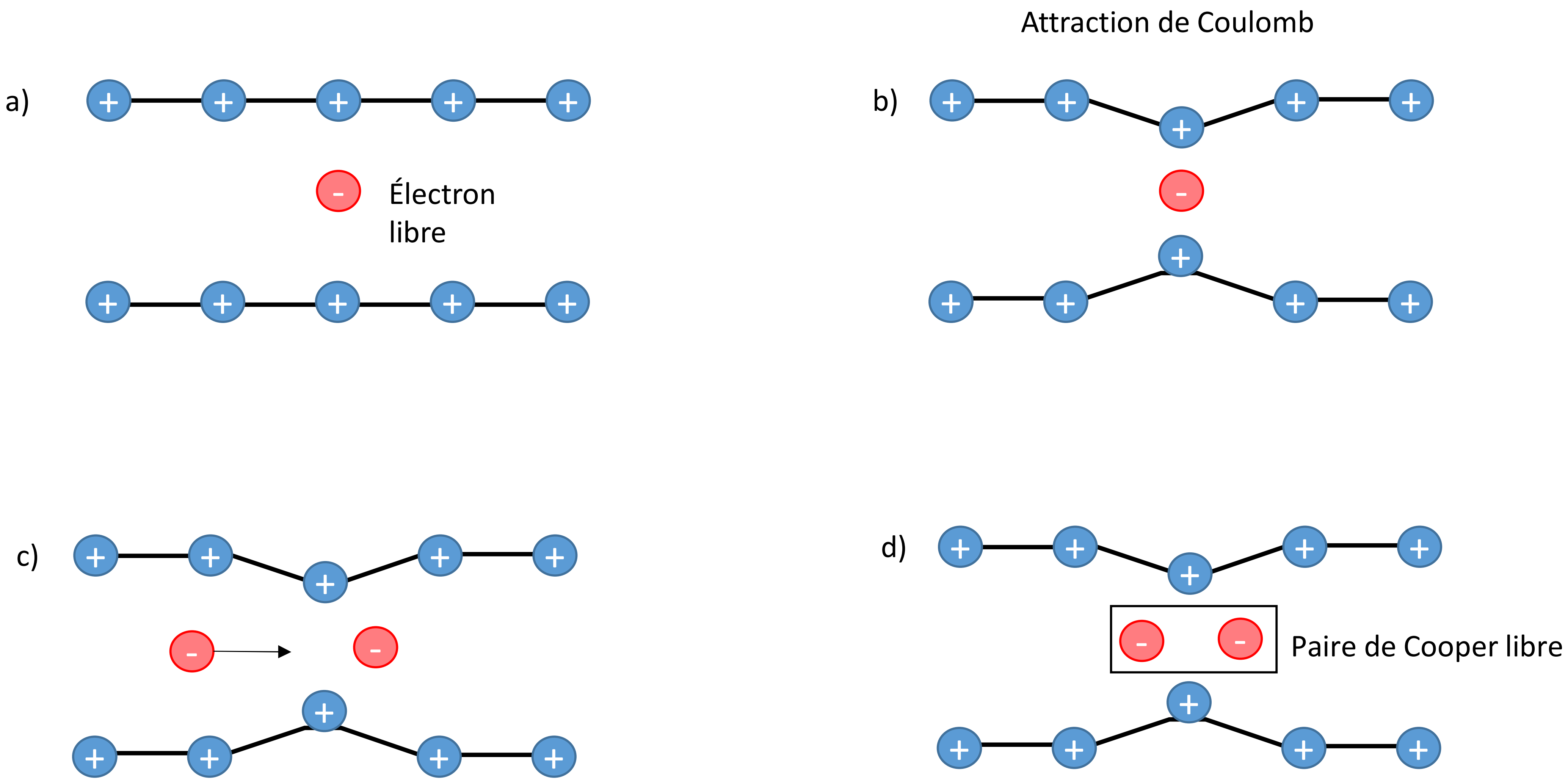
Jonction de Josephson: structure

J.C Besse, ETH Thesis 27386

T<100mK

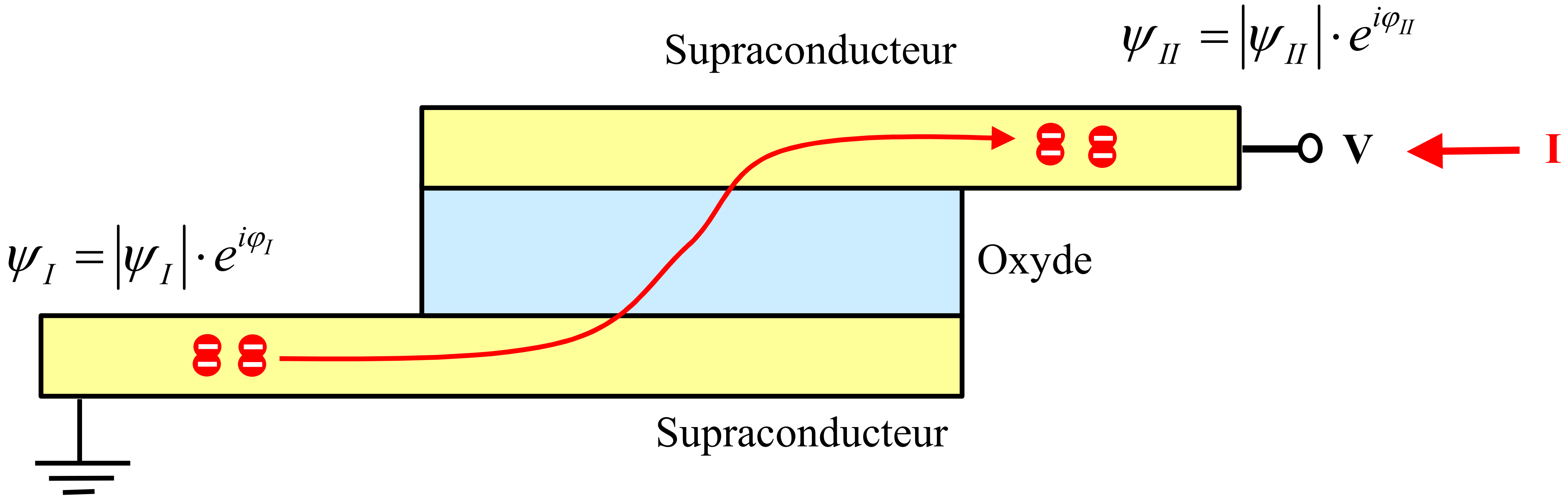


Supraconductivité: formation de paires de Cooper

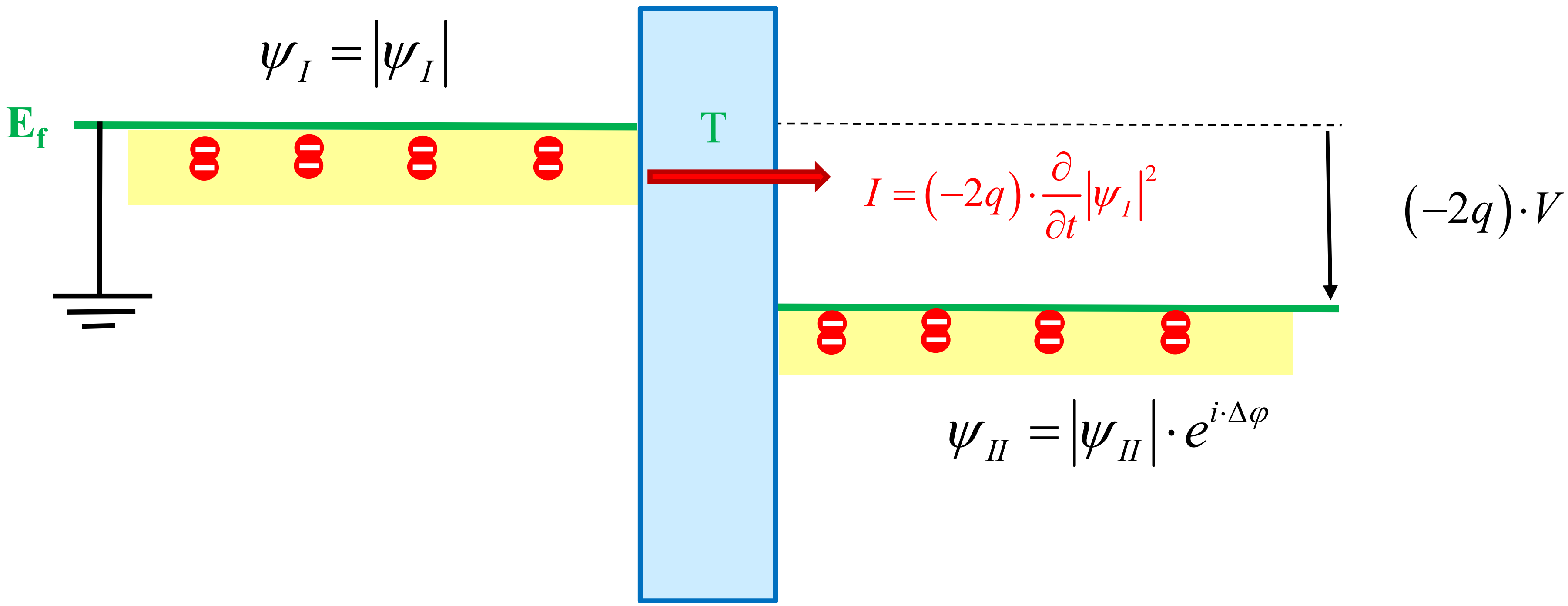


Jonction de Josephson: principe

Effet tunnel pour des paires de Cooper à des températures de mK.



Modèle:
effet tunnel entre deux lacs de paires de Cooper



$$H = \begin{pmatrix} 0 & -T \\ -T & -2qV \end{pmatrix}$$

Jonction Josephson = coupleur par effet tunnel

Josephson junction:

$$i\hbar \cdot \frac{\partial}{\partial t}\begin{pmatrix}\psi_I \\ \psi_{II}\end{pmatrix} = \begin{pmatrix}0 & -T \\ -T & -2qV\end{pmatrix}\cdot\begin{pmatrix}\psi_I \\ \psi_{II}\end{pmatrix}$$

Supposons:

$$\psi_I = |\psi_I| \cdot e^{i\varphi_I} \quad \Longrightarrow \quad i\hbar \cdot \frac{\partial}{\partial t}\psi_I = i\hbar \cdot e^{i\varphi_I} \cdot \frac{\partial}{\partial t}|\psi_I| + i\hbar \cdot |\psi_I| \cdot e^{i\varphi_I} \cdot i\frac{\partial}{\partial t}\varphi_I$$

$$\psi_{II} = |\psi_{II}| \cdot e^{i\varphi_{II}} \quad \Longrightarrow \quad i\hbar \cdot \frac{\partial}{\partial t}\psi_{II} = i\hbar \cdot e^{i\varphi_{II}} \cdot \frac{\partial}{\partial t}|\psi_{II}| + i\hbar \cdot |\psi_{II}| \cdot e^{i\varphi_{II}} \cdot i\frac{\partial}{\partial t}\varphi_{II}$$

$$\Longrightarrow \left\{\begin{array}{l} \frac{\partial}{\partial t}|\psi_I| + i \cdot |\psi_I| \cdot \frac{\partial}{\partial t}\varphi_I = \frac{T}{\hbar}|\psi_{II}|\left(i \cdot \cos(\varphi_{II} - \varphi_I) - \sin(\varphi_{II} - \varphi_I)\right) \\ \frac{\partial}{\partial t}|\psi_{II}| + i \cdot |\psi_{II}| \cdot \frac{\partial}{\partial t}\varphi_{II} = i \cdot \frac{2qV}{\hbar} \cdot |\psi_{II}| + \frac{T}{\hbar}|\psi_I|\left(i \cdot \cos(\varphi_I - \varphi_{II}) - \sin(\varphi_I - \varphi_{II})\right) \end{array}\right.$$

Supposons: $|\psi_I| \cong |\psi_{II}| \equiv |\psi|$

Partie réelle:

$$\begin{cases} \frac{\partial}{\partial t}|\psi_I| = -\frac{T}{\hbar}|\psi|\cdot\sin(\varphi_{II}-\varphi_I) \\ \frac{\partial}{\partial t}|\psi_{II}| = \frac{T}{\hbar}|\psi|\cdot\sin(\varphi_{II}-\varphi_I) \end{cases} \Longrightarrow$$

Courant de II vers I

$$I = (-2q)\cdot\frac{\partial}{\partial t}|\psi_I|^2 = \underbrace{\left[4q\frac{T}{\hbar}|\psi|^2\right]}_{\mathbf{I_0}}\cdot\sin(\Delta\varphi)$$

$\varphi_{II}-\varphi_I$

$$I = I_0\cdot\sin(\Delta\varphi)$$

Partie imaginaire:

$$\begin{cases} \frac{\partial}{\partial t}\varphi_I = \frac{T}{\hbar}\cos(\Delta\varphi) \\ \frac{\partial}{\partial t}\varphi_{II} = \frac{2qV}{\hbar}+\frac{T}{\hbar}\cos(\Delta\varphi) \end{cases} \Longrightarrow$$

$$V = \frac{\hbar}{(2q)}\cdot\frac{\partial}{\partial t}\Delta\varphi$$

Lois de Josephson:

$$I = I_0 \cdot \sin(\Delta\varphi)$$

$$V = \frac{\hbar}{(2q)} \cdot \frac{\partial}{\partial t} \Delta\varphi$$

Inductance de Josephson:

$$\frac{\partial I}{\partial t} = I_0 \cos(\Delta\varphi) \cdot \frac{\partial}{\partial t} \Delta\varphi = \frac{(2q)}{\hbar} I_0 \cos(\Delta\varphi) \cdot V$$

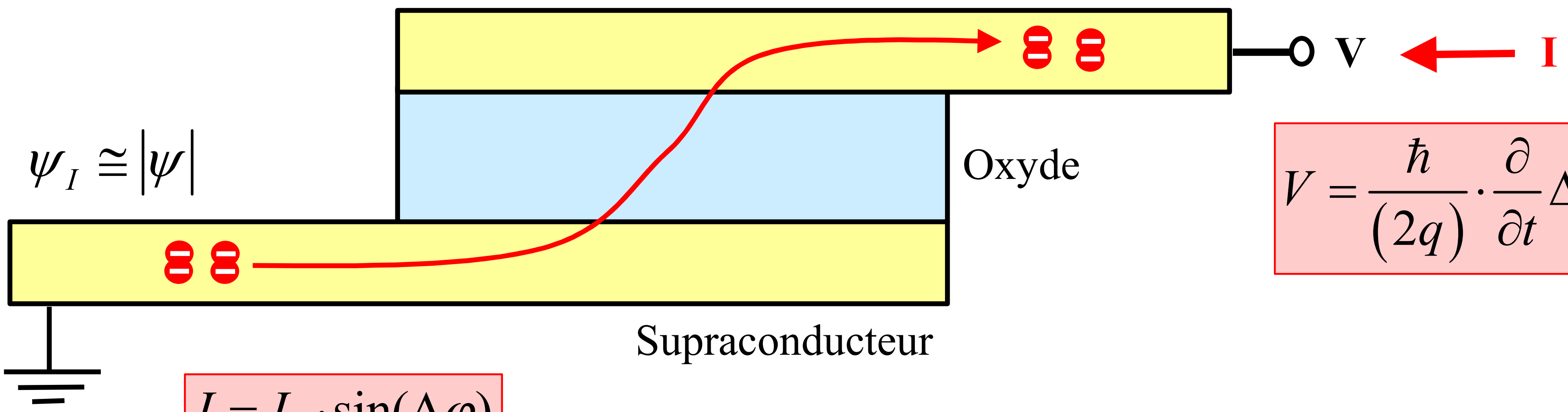
$$\frac{\partial I}{\partial t} \equiv \frac{1}{L_J} \cdot V \quad \Longrightarrow \quad \frac{1}{L_J} = \frac{1}{L_0} \cdot \cos(\Delta\varphi) \qquad \frac{1}{L_0} \equiv \frac{2qI_0}{\hbar}$$

Energie de Josephson:

$$E_J \equiv \int V \cdot I \cdot dt = \frac{I_o \hbar}{2q} \int_0^{\Delta\varphi} \sin(\varphi)\, d\varphi \quad \Rightarrow \quad E_J = \frac{I_o \hbar}{2q}\left(1 - \cos(\Delta\varphi)\right)$$

Jonction de Josephson: résumé

Effet tunnel pour des paires de Cooper à des températures de mK.



$$\psi_{II} \cong |\psi| \cdot e^{i\Delta\varphi}$$

$$\psi_I \cong |\psi|$$

$$V = \frac{\hbar}{(2q)} \cdot \frac{\partial}{\partial t} \Delta\varphi$$

$$I = I_0 \cdot \sin(\Delta\varphi)$$

$$I_0 \equiv 4q \frac{T}{\hbar} |\psi|^2$$

$$\frac{1}{L_J} = \frac{1}{L_0} \cdot |\cos(\Delta\varphi)|$$

$$E_J = \frac{I_o \hbar}{2q} (1 - \cos(\Delta\varphi))$$

$$\frac{1}{L_0} \equiv \frac{2qI_0}{\hbar}$$

Hamiltonien «inductif» d'une jonction de Josephson

anharmonique

Pour une jonction de Josephson:

$$H_J \equiv E_J = \frac{I_o \hbar}{2q}\left(1-\cos(\Delta\varphi)\right) \cong \frac{1}{2}\frac{1}{L_0}\cdot\left(\frac{\hbar}{2q}\right)^2 \Delta\varphi^2 - O\left(\Delta\varphi^4\right)$$

Pour une bobine inductive supraconductrice:

Flux magnétique

$$H_J = \frac{1}{2}\frac{1}{L}\cdot\phi_{mag}^2 = \frac{1}{2}\frac{1}{L}\cdot\left(\frac{\hbar}{2q}\right)^2 \Delta\varphi^2$$

Phase de Berry:

$$\Delta\varphi = \frac{(2q)}{\hbar}\cdot\oint \vec{A}\cdot d\vec{l} = 2\pi\cdot\frac{\phi_{mag}}{\phi_0}$$

$$\phi_0 \equiv \text{flux quantum} \equiv \frac{h}{(2q)}$$

Couplage capacitif

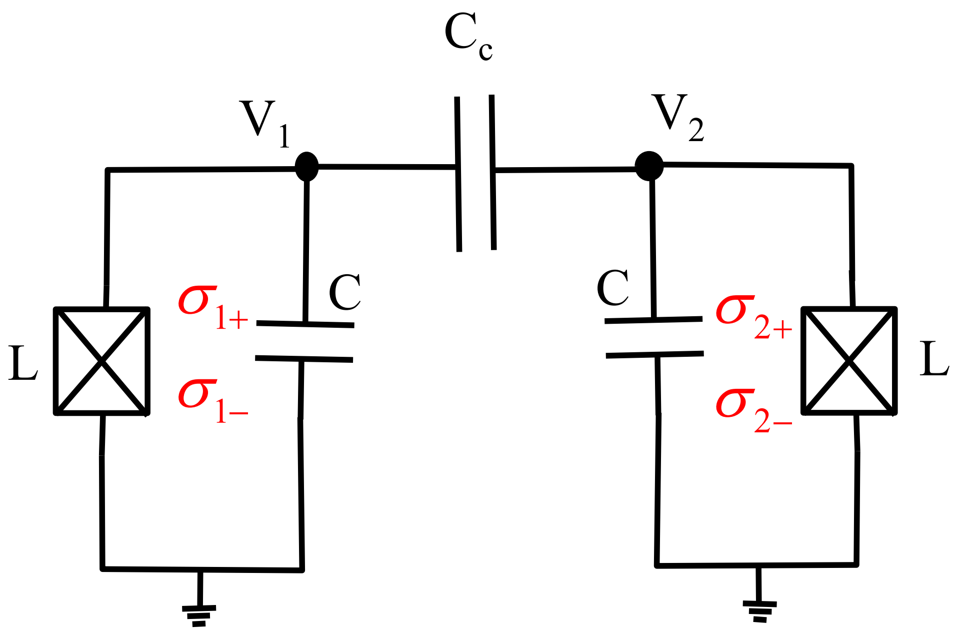
de deux qubits:

iSWAP

Couplage capacitif de deux qubits

$$H_c \approx -T \cdot \left(\sigma_{1+}\sigma_{2-} + \sigma_{1-}\sigma_{2+} \right)$$

$$H_c \approx -T \cdot \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



Deux Qubits dans leur référentiel fixe

Référentiel fixe:

Qubit «R» $|\psi_R\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$

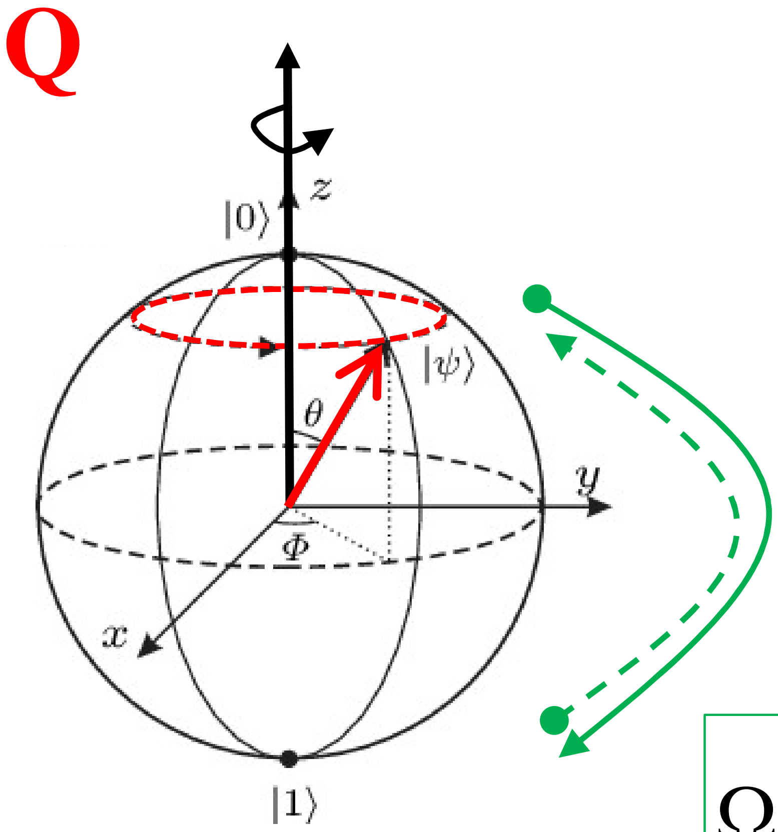
Qubit «Q» $|\psi_Q\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$

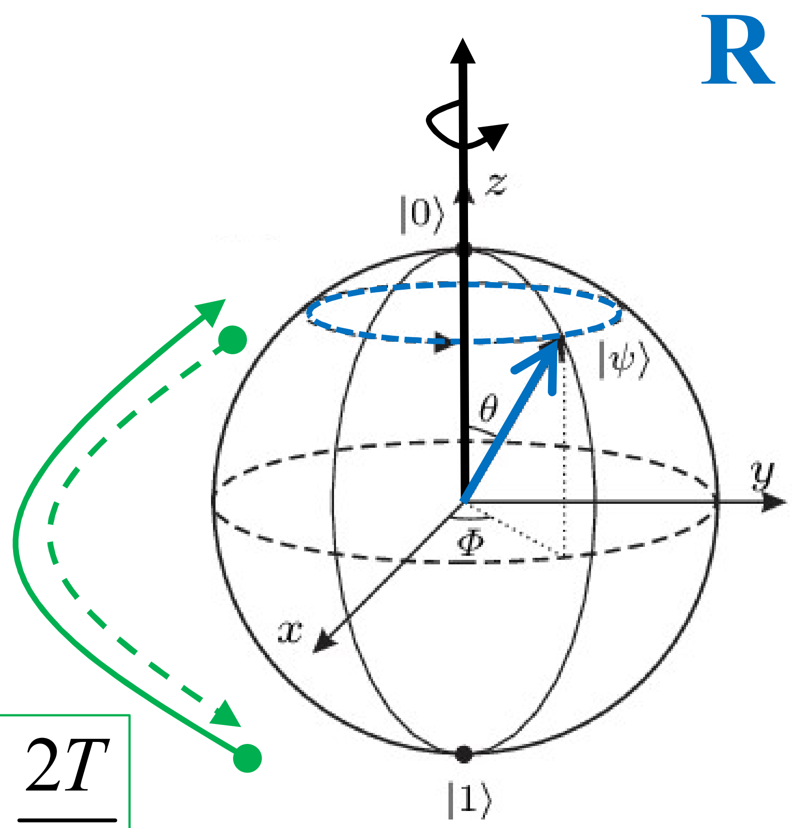
$\Longrightarrow$ $|\psi\rangle = \begin{pmatrix} a_0\,\alpha_0 \\ a_0\,\alpha_1 \\ a_1\,\alpha_0 \\ a_1\,\alpha_1 \end{pmatrix}$

$$i\hbar \frac{\partial}{\partial t}|\psi\rangle = H \cdot |\psi\rangle$$

$$H = H_R + H_Q + H_C = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & E_Q & -T & 0 \\ 0 & -T & E_R & 0 \\ 0 & 0 & 0 & E_Q + E_R \end{pmatrix}$$

Deux qubits couplés



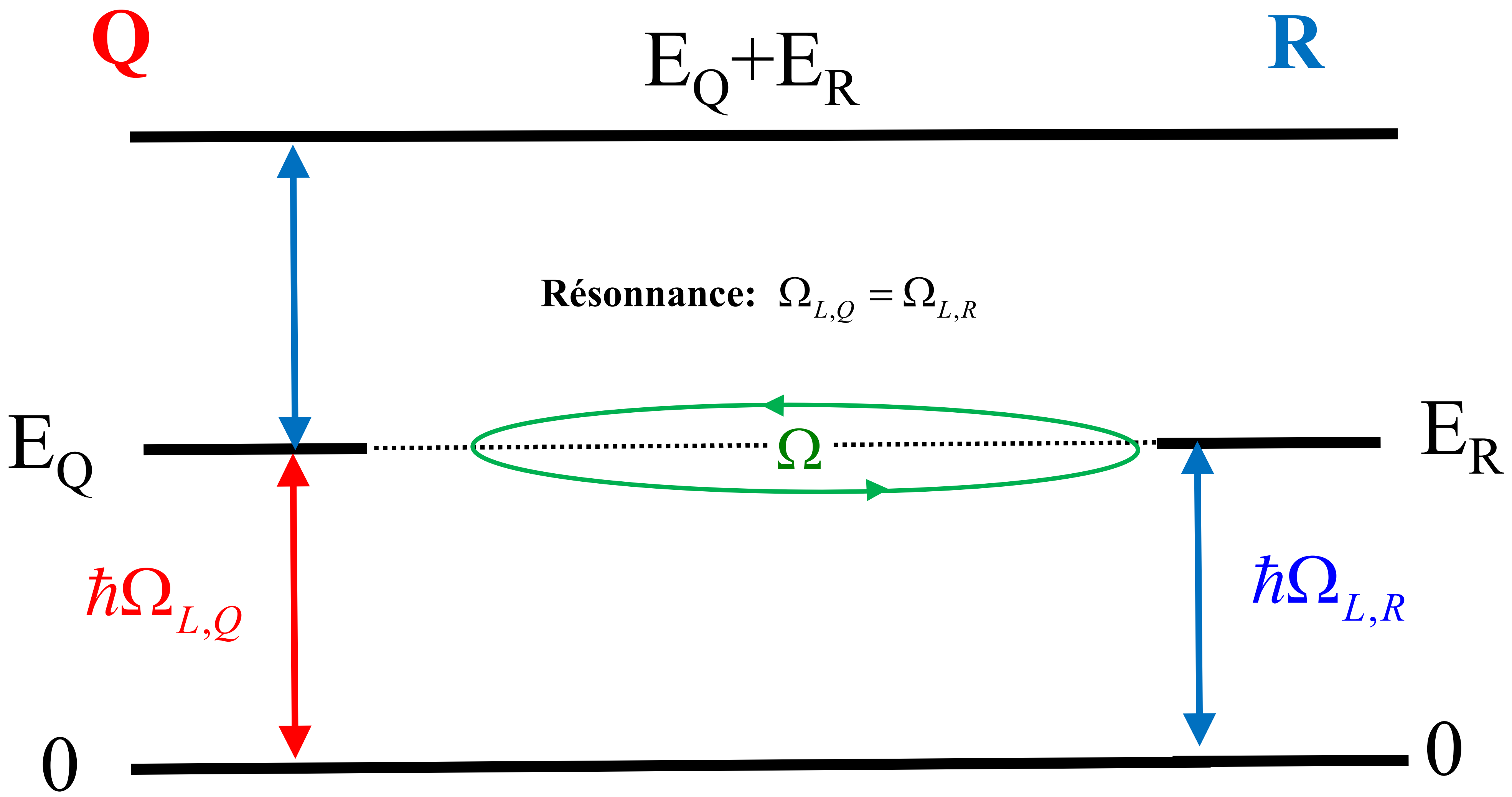


$$\Omega \equiv \frac{2T}{\hbar}$$

$$\Omega_{L,Q} \equiv \frac{E_Q}{\hbar}$$

$$\Omega_{L,R} \equiv \frac{E_R}{\hbar}$$

Deux qubits couplés



En résonnance:

Double Qubit:

Référentiel fixe

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & E_Q & -T & 0 \\ 0 & -T & E_R & 0 \\ 0 & 0 & 0 & E_Q + E_R \end{pmatrix}$$

$$\hbar\omega = E_R = E_Q \quad \Rightarrow$$

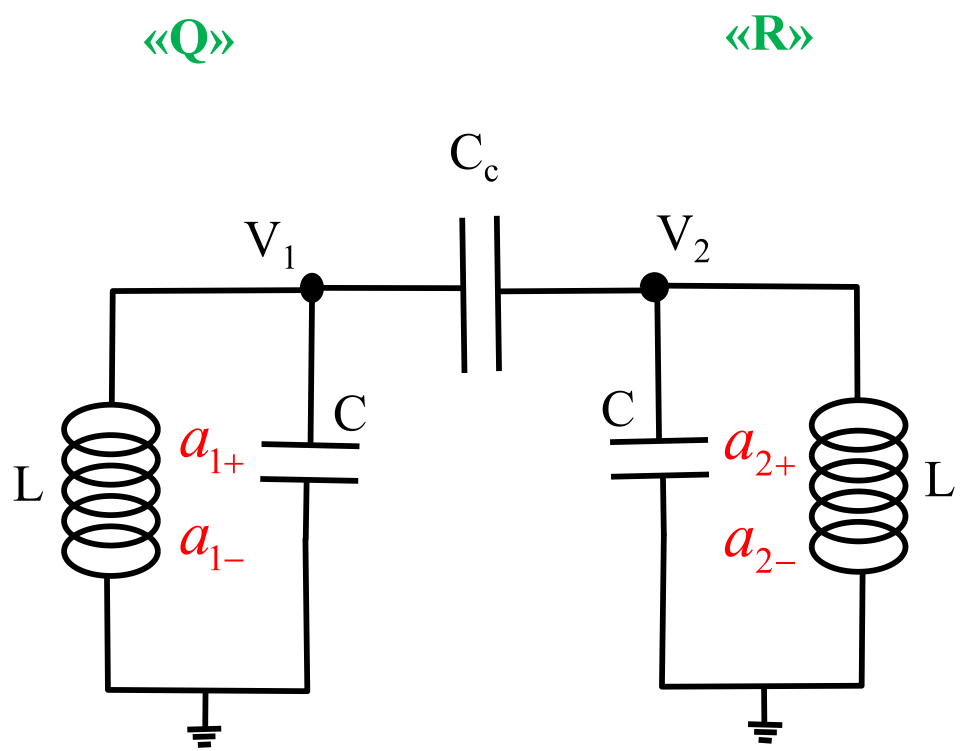
Référentiel tournant en résonnance

$$\bar{H} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -T & 0 \\ 0 & -T & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Couplage capacitif

Qubit / résonateur

Rappel: couplage de deux résonnateurs «Q» et «R»



$$H_c \approx -T \cdot \left(a_{1+} a_{2-} + a_{1-} a_{2+} \right)$$

$$a_+ = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{n} & 0 \end{pmatrix}$$

$$a_- = \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{n} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Rappel:
Couplage capacitif de deux qubits (2x2)

$$H \equiv \left(H_R + H_Q\right) + H_C = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & E_Q & -\sqrt{1\cdot 1}\cdot T & 0 \\ 0 & -\sqrt{1\cdot 1}\cdot T & E_R & 0 \\ 0 & 0 & 0 & E_R + E_Q \end{pmatrix}$$

$$|\psi\rangle = \begin{pmatrix} |0\rangle_R |0\rangle_Q \\ |0\rangle_R |1\rangle_Q \\ |1\rangle_R |0\rangle_Q \\ |1\rangle_R |1\rangle_Q \end{pmatrix}$$

$$E_{00} = 0$$

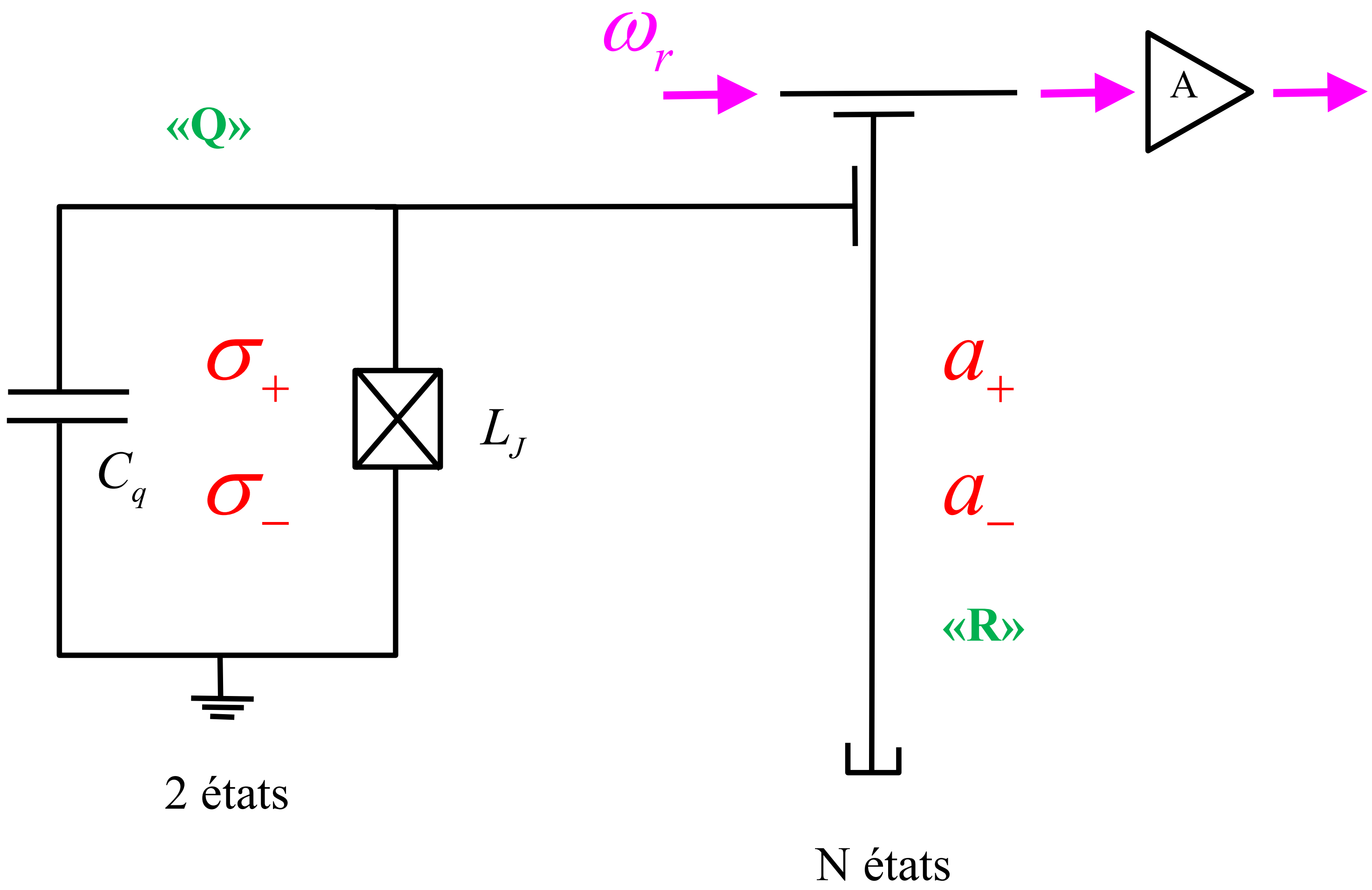
R Q

$$E_{01} = \left(\frac{E_R + E_Q}{2}\right) + \sqrt{\left(\frac{E_Q - E_R}{2}\right)^2 + T^2}$$

$$E_{10} = \left(\frac{E_R + E_Q}{2}\right) - \sqrt{\left(\frac{E_Q - E_R}{2}\right)^2 + T^2}$$

$$E_{11} = E_R + E_Q$$

Transmon et résonateur couplés:



$$\overline{E}_{n,m} = n \cdot E_R + m \cdot E_Q \qquad n = 0,\ldots N \qquad et \qquad m = 0,1$$

PAB

Couplage résonateur/Qubit (Nx2)

$$\overline{E}_{n,m} = n \cdot E_R + m \cdot E_Q \qquad n = 0,\ldots N \quad m = 0,1$$

$$H = \begin{pmatrix} \boxed{0} & \cdots & & & & 0 \\ \cdots & \boxed{\begin{matrix} E_Q & -T \\ -T & E_R \end{matrix}} & 0 & & & \cdots \\ & 0 & \boxed{\begin{matrix} \cdots & \cdots \\ \cdots & \cdots \end{matrix}} & & & \\ & & & \boxed{\begin{matrix} (n-1)E_R + E_Q & -T \cdot a_- \sigma_+ \\ -T \cdot a_+ \sigma_- & n \cdot E_R \end{matrix}} & & \\ & & & & \boxed{\begin{matrix} \cdots & \cdots \\ \cdots & \cdots \end{matrix}} & 0 \\ 0 & \cdots & & & 0 & \boxed{(N-1)E_R + E_Q} \end{pmatrix} \qquad |\psi\rangle = \begin{pmatrix} |0\rangle_R |0\rangle_Q \\ |0\rangle_R |1\rangle_Q \\ |1\rangle_R |0\rangle_Q \\ \cdots \\ \cdots \\ |n-1\rangle_R |1\rangle_Q \\ |n\rangle_R |0\rangle_Q \\ \cdots \\ \cdots \\ |N-1\rangle_R |1\rangle_Q \end{pmatrix}$$

$$E_{\substack{0,1 \\ 1,0}} = \left(\frac{E_R + E_Q}{2} \right) \pm \sqrt{\left(\frac{E_Q - E_R}{2} \right)^2 + T^2}$$

$$E_{0,0} = 0$$

$$E_{\substack{(n-1),1 \\ n,0}} = \left(\frac{(2n-1)E_R + E_Q}{2} \right) \pm \sqrt{\left(\frac{E_Q - E_R}{2} \right)^2 + (\sqrt{n} \cdot T)^2}$$

$$E_{(N-1),1} = (N-1)E_R + E_Q$$

Couplage résonateur/Qubit (Nx2)

$$\begin{pmatrix} \cdots & 0 & 0 \\ 0 & \boxed{\begin{matrix} (n-1)E_R + E_Q & -T\cdot\sqrt{n\cdot 1} \\ -T\cdot\sqrt{n\cdot 1} & n\cdot E_R \end{matrix}} & 0 \\ 0 & 0 & \cdots \end{pmatrix} \cdot \begin{pmatrix} \cdots \\ |n-1\rangle_R\,|1\rangle_Q \\ |n\rangle_R\,|0\rangle_Q \\ \cdots \end{pmatrix}$$

$$\Downarrow$$

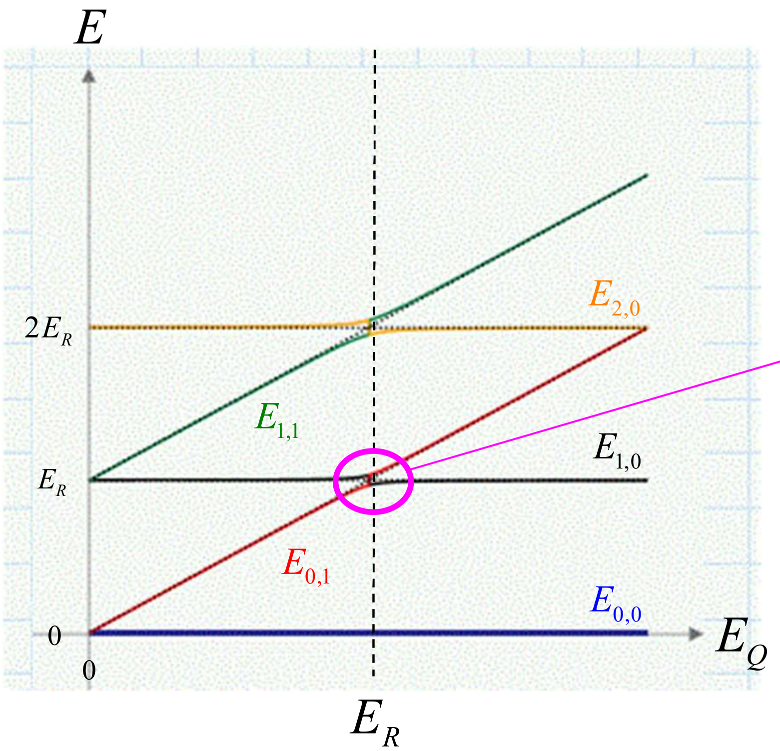
$$E_{(n-1),1} = \left(\frac{(2n-1)E_R + E_Q}{2}\right) + \sqrt{\left(\frac{E_Q - E_R}{2}\right)^2 + \left(\sqrt{n}\,T\right)^2}$$

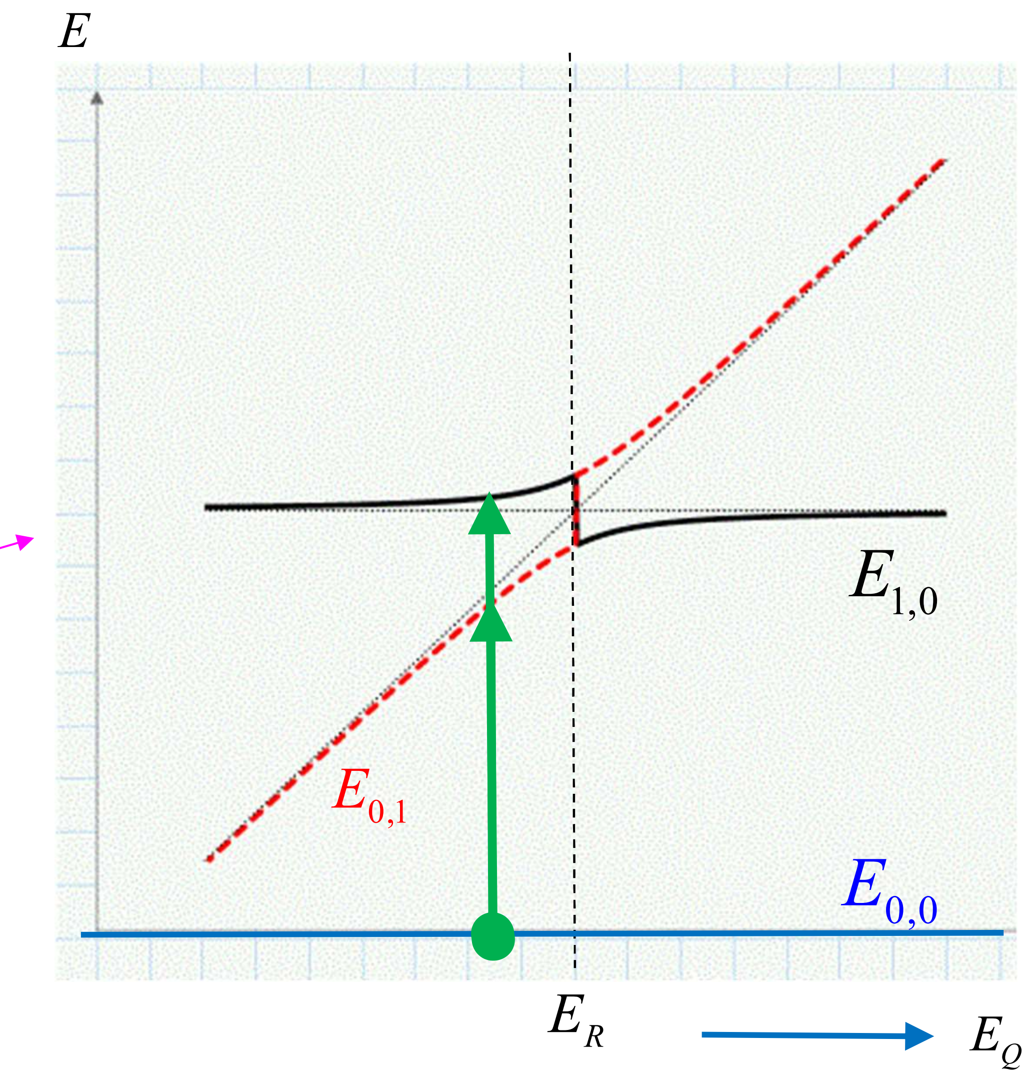
$$E_{n,0} = \left(\frac{(2n-1)E_R + E_Q}{2}\right) - \sqrt{\left(\frac{E_Q - E_R}{2}\right)^2 + \left(\sqrt{n}\,T\right)^2}$$

Solutions en résonance

Résonance: $\dfrac{|E_Q - E_R|}{T} \ll 1$

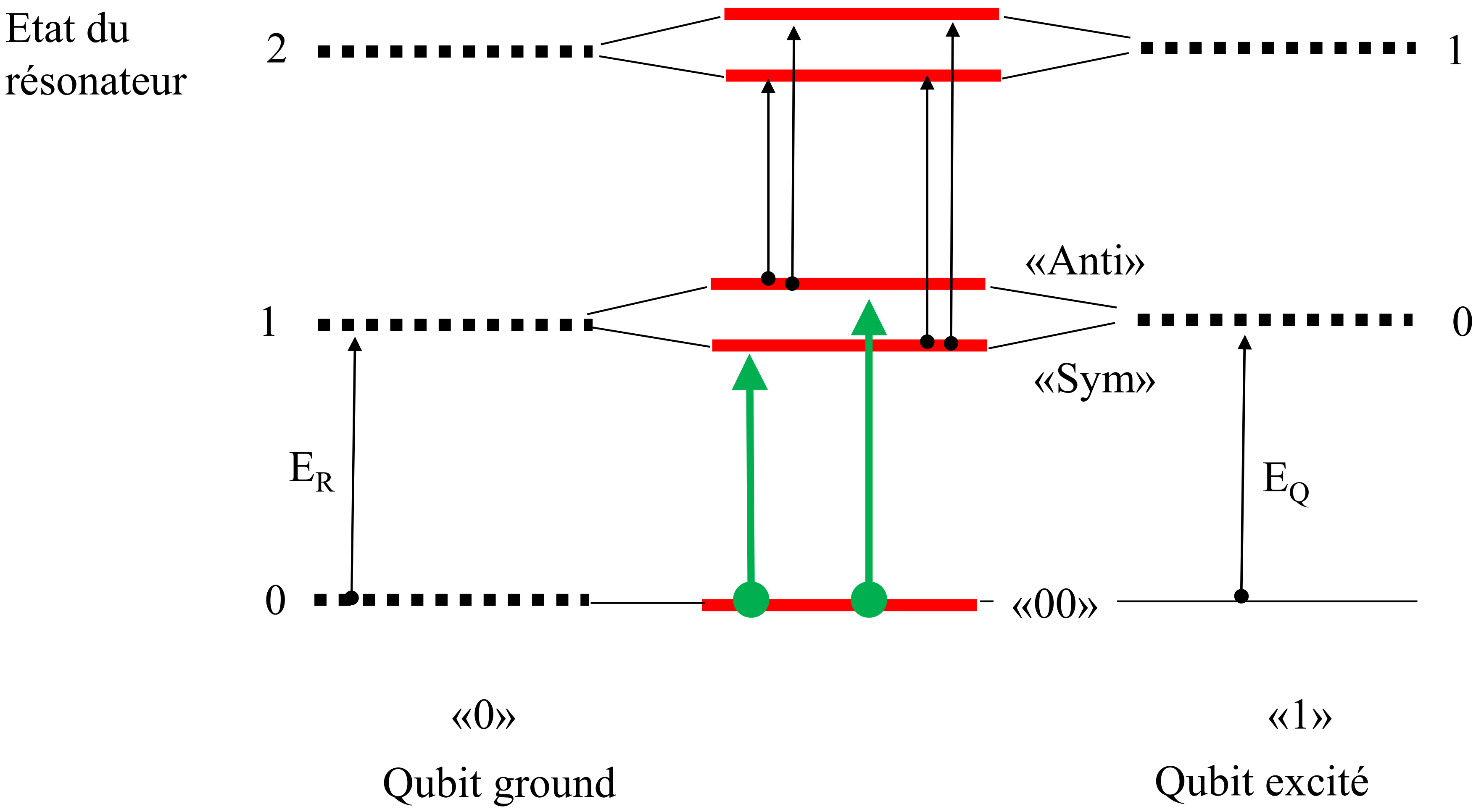
Couplage des modes





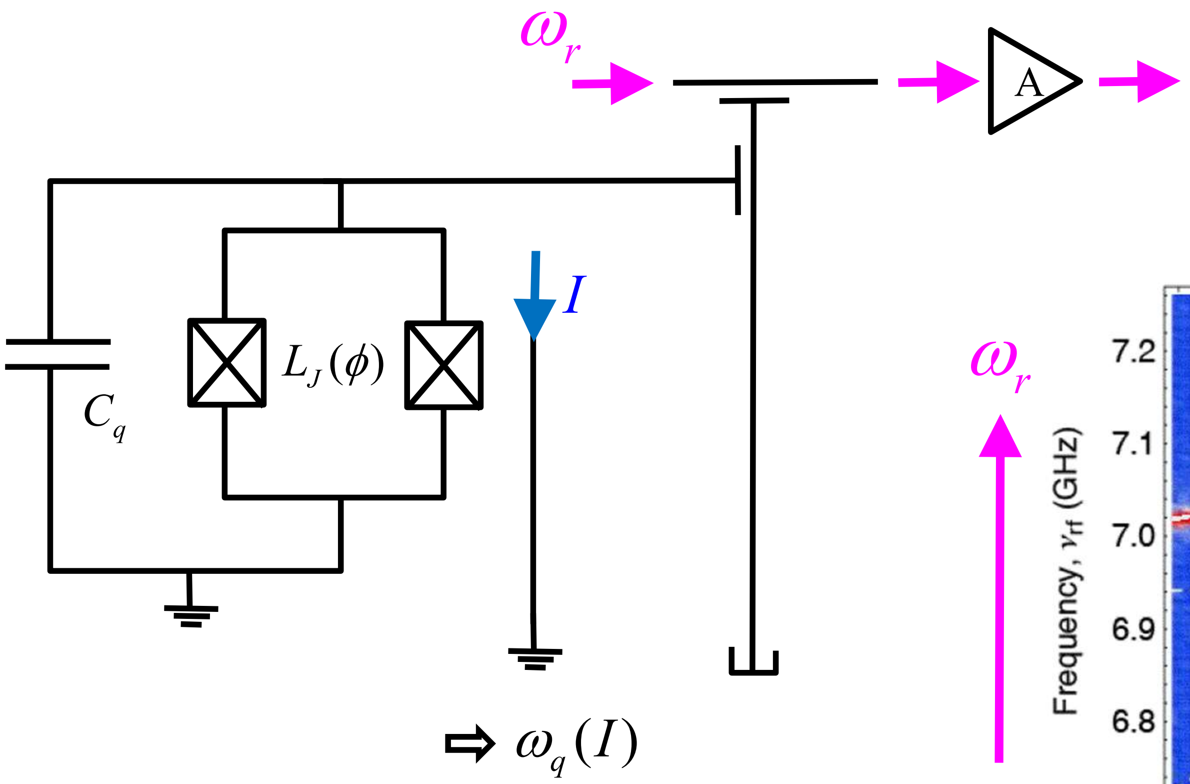
Solutions en résonance $E_Q = E_R$

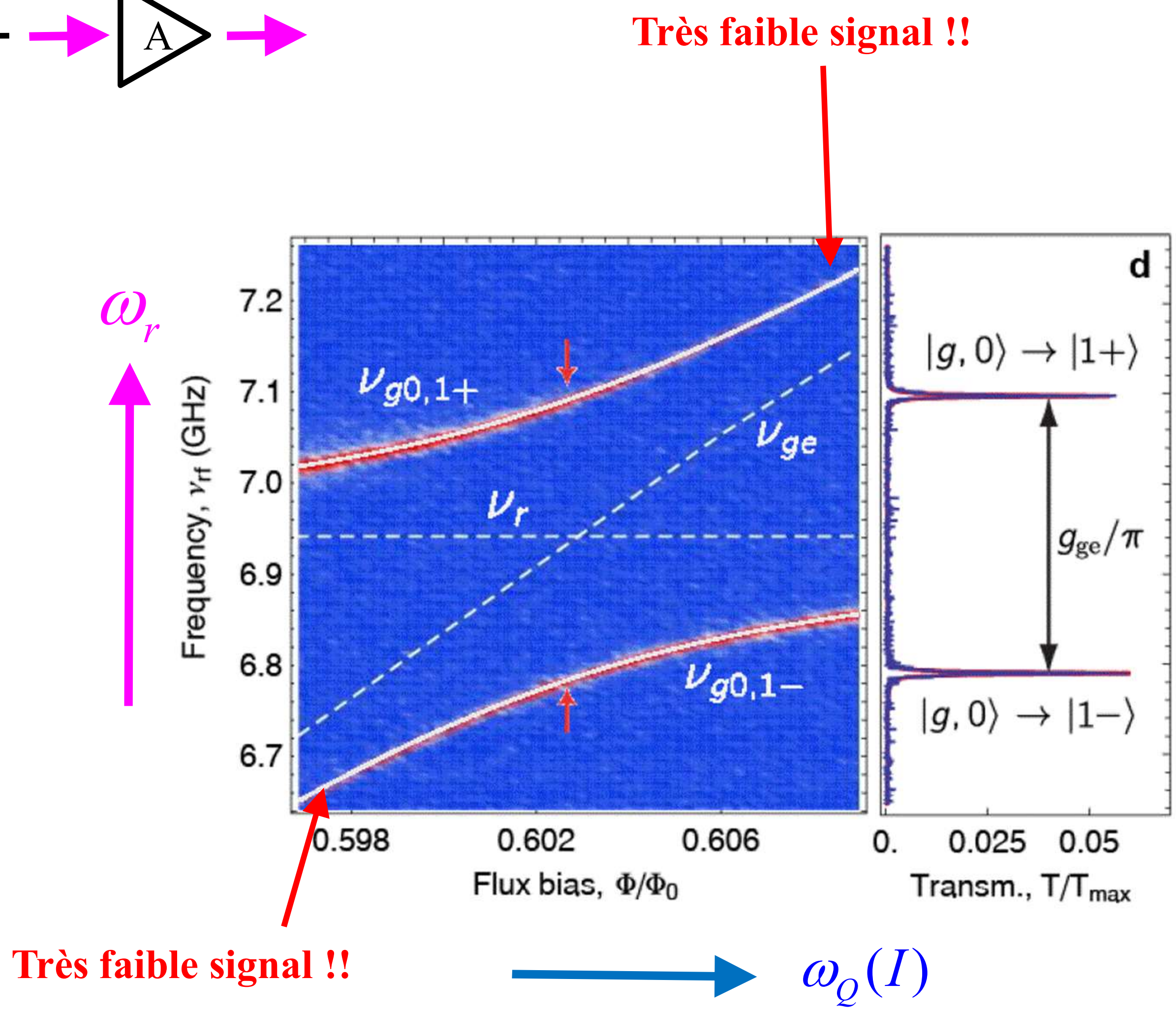
Le Qubit n'est pas excité → transition «00-Sym» ou «00-Anti»



Transmon et résonateur couplés: Mesures en résonnance

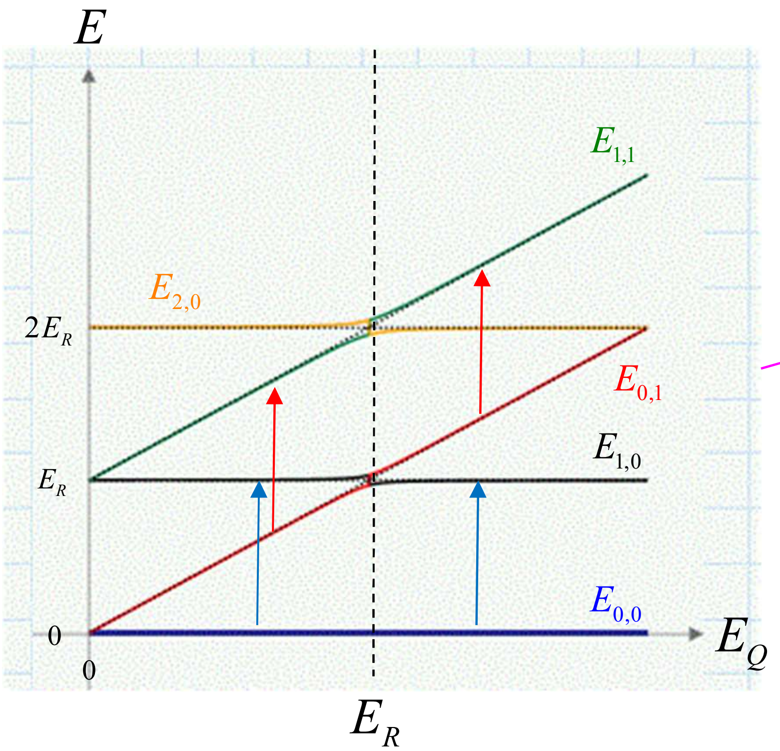
J. Fink et al., *Nature (London)* **454**, 315 (2008)



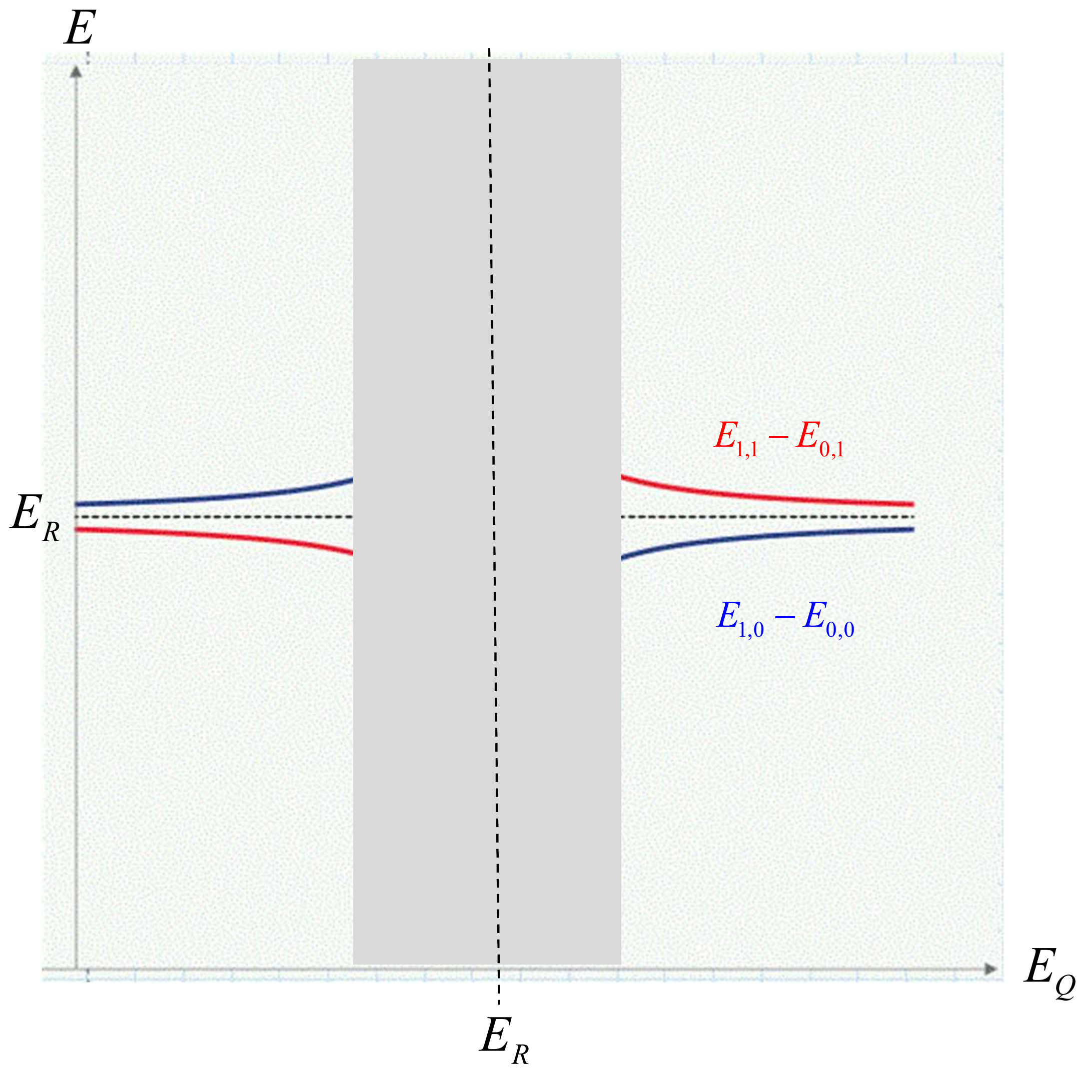


Chaque point de la figure correspond à une moyenne sur env. 10'000 mesures

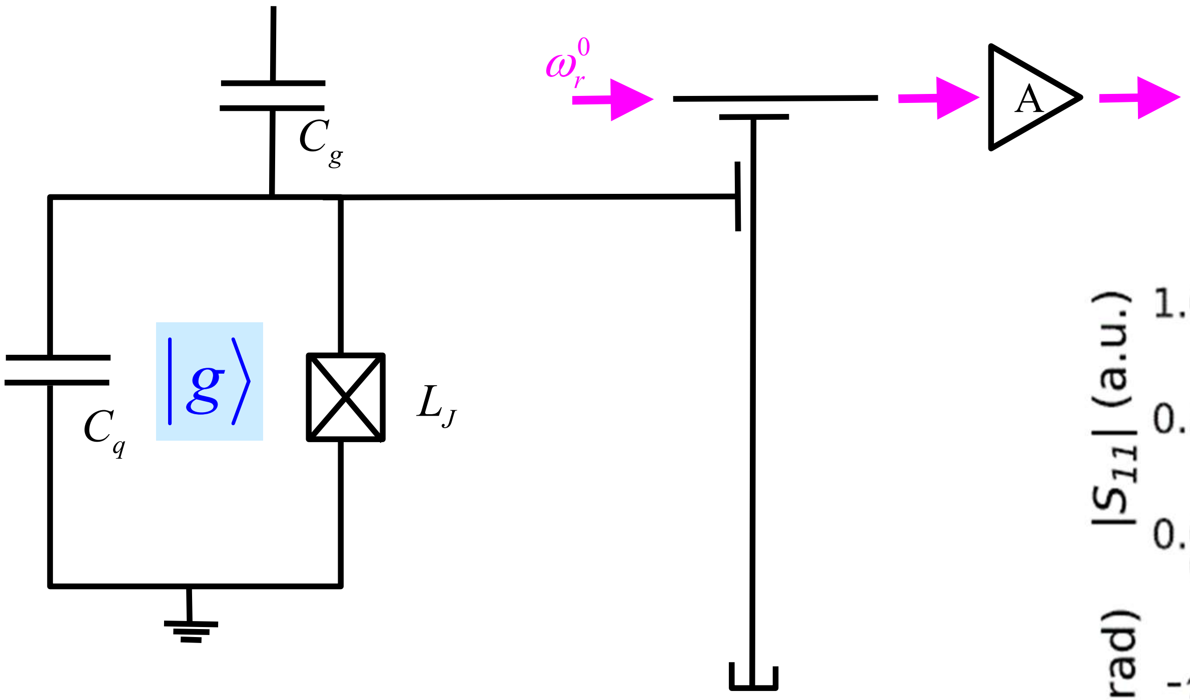
Solutions en mode dispersif $\frac{|E_Q - E_R|}{T} \gg 1$



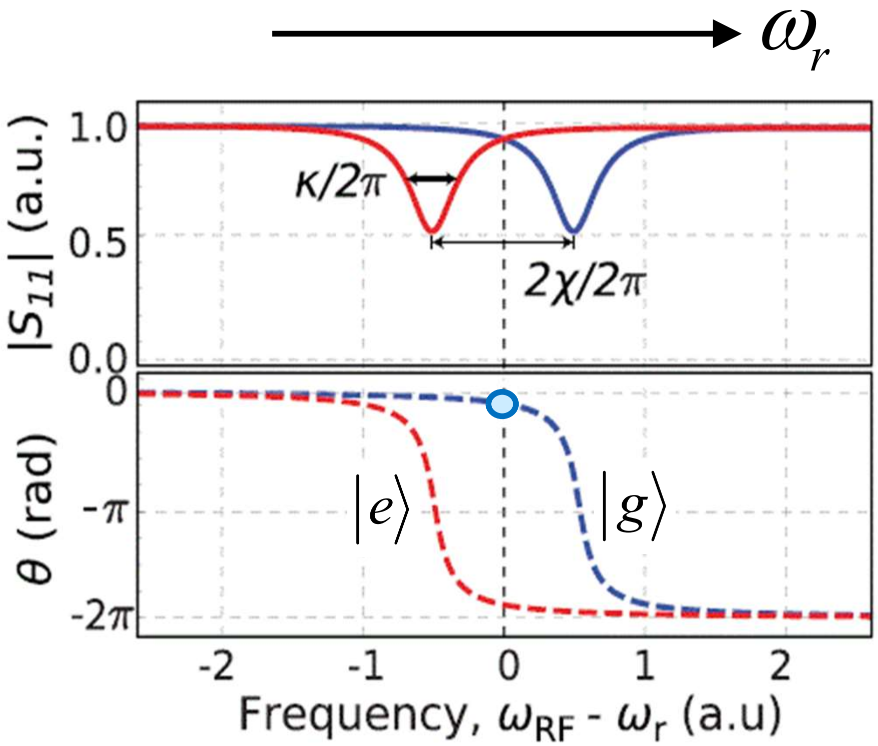
Shift de la fréquence du résonateur



Transmon et résonateur en régime dispersif

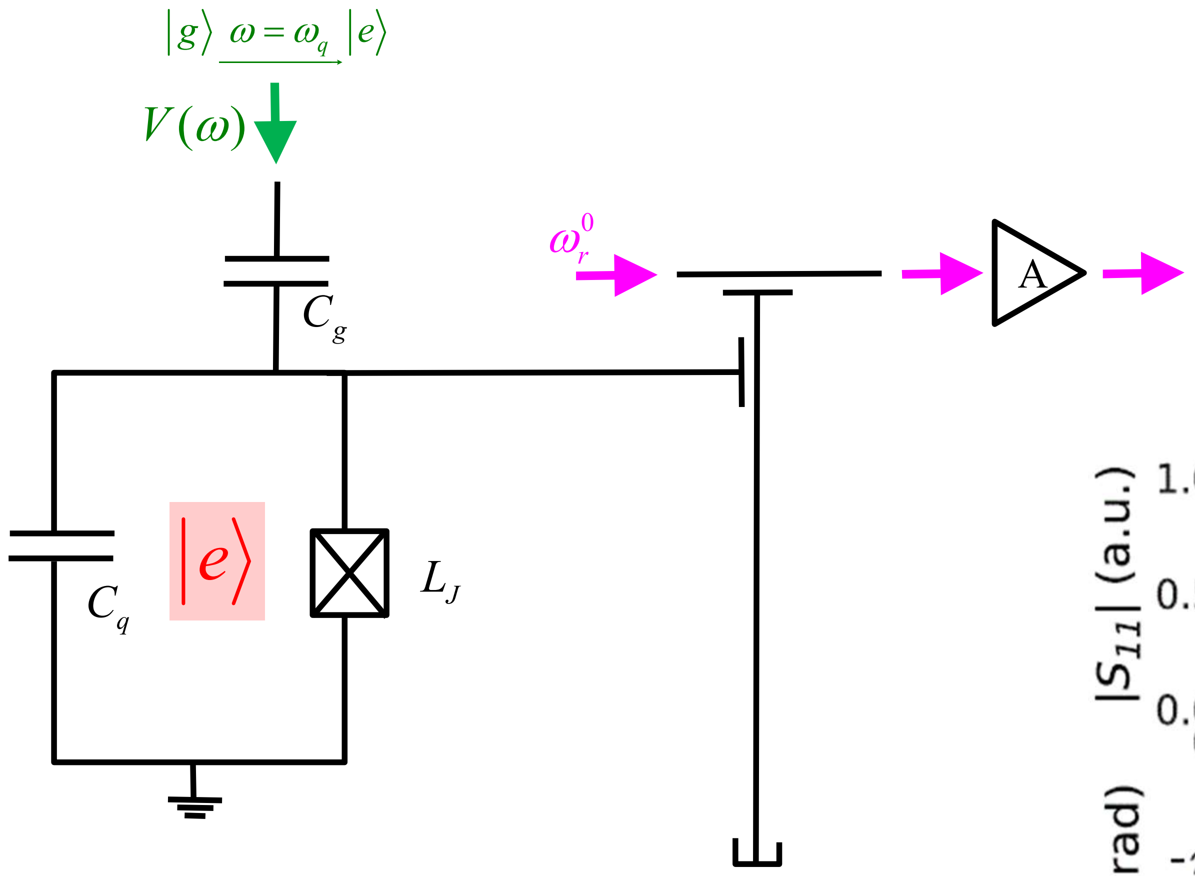


«dispersive shift»

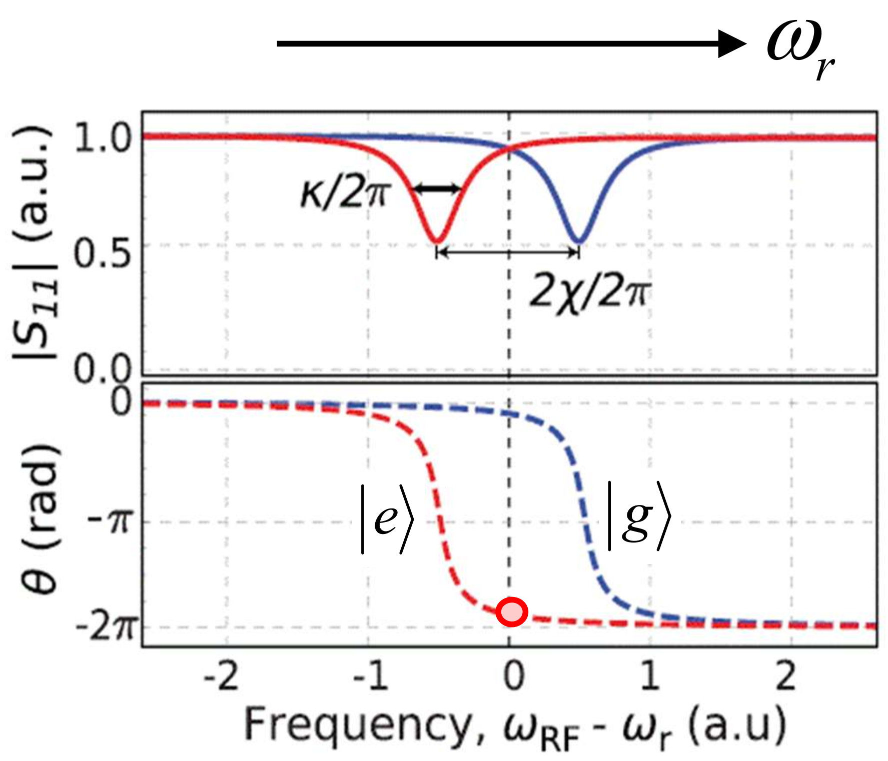


«non-demolition measurement»

Transmon et résonateur en régime dispersif



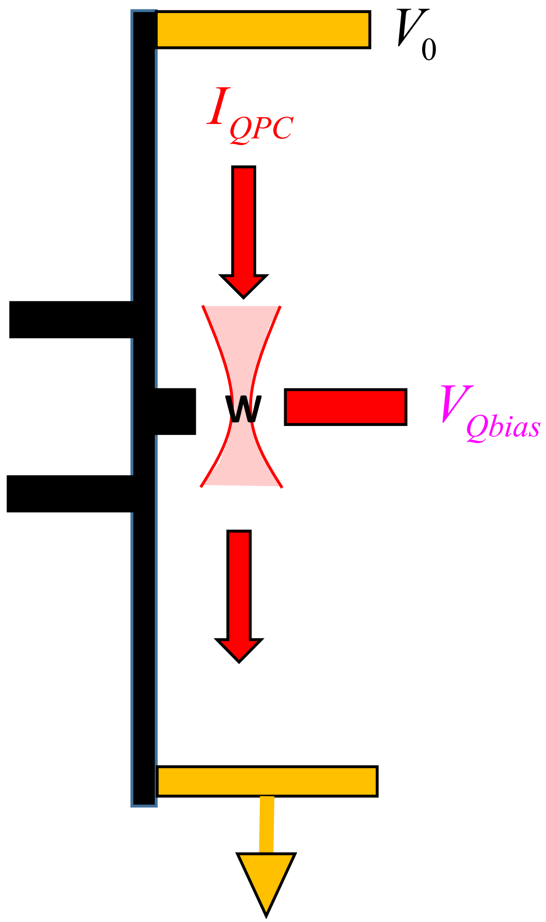
«dispersive shift»



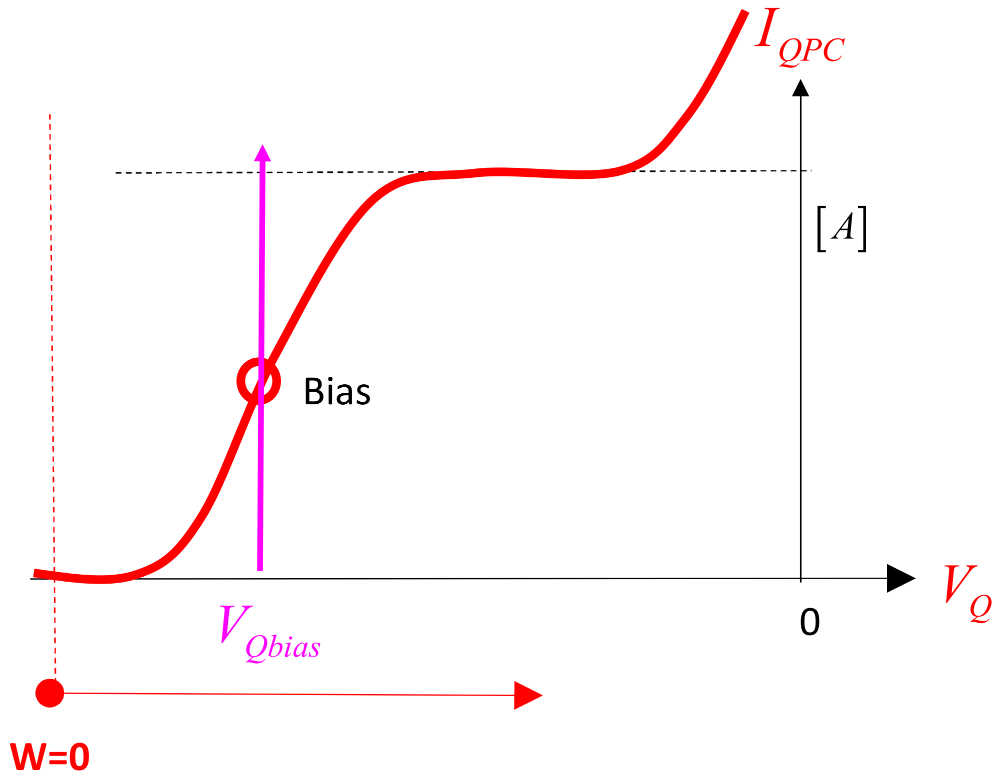
«non-demolition measurement»

« non-demolition measurement» Rappel: Quantum Point Contact

QPC



Courant



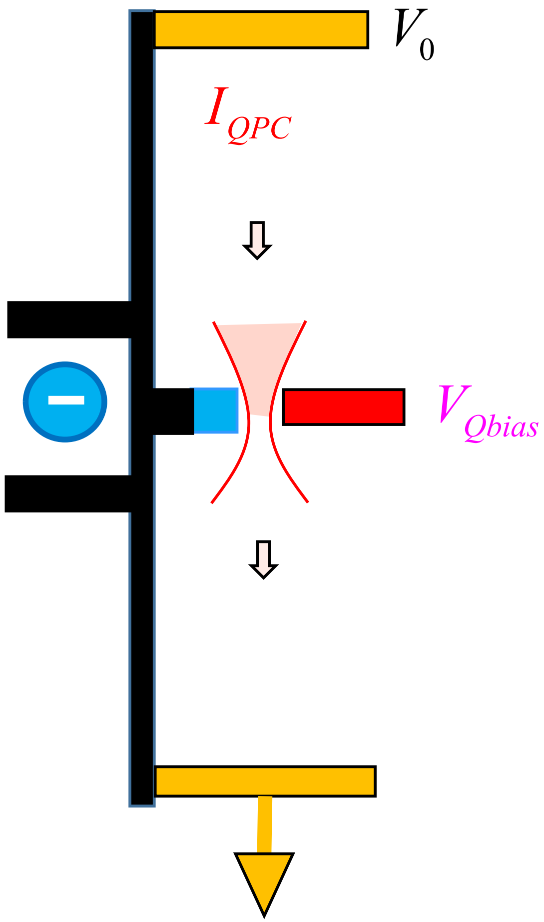
Courant:

$$I_{QPC} = G\left(V_Q\right) \cdot V_0$$

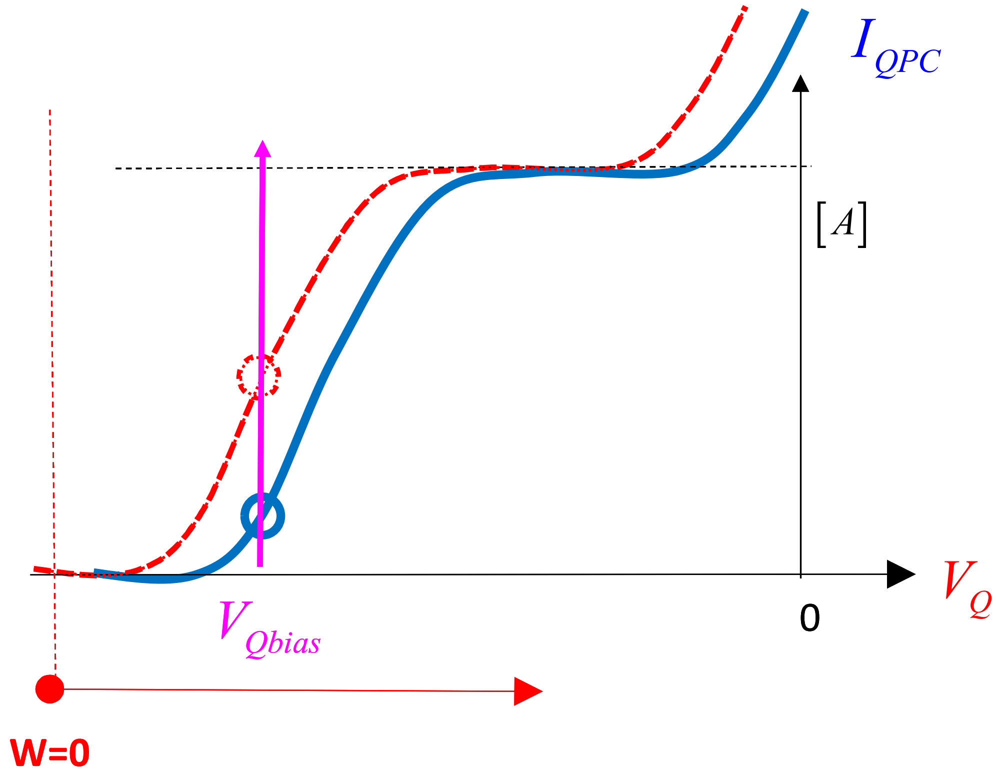
fixe

« non-demolition measurement»
Rappel: Quantum Point Contact

QPC



Courant



Courant: $$I_{QPC} = G\left(V_Q, N_e\right) \cdot V_0$$

fixe

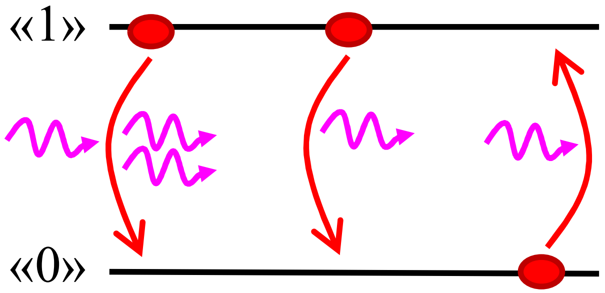
Mesure «non-démolition»

Interaction

lumière – matière

Absorption, émission

Absorption, émission
LED



$$H_{ém} \approx \beta \cdot a_{+} \sigma_{-}$$

photon électron

$$H_{ab} \approx \beta \cdot a_{-} \sigma_{+}$$

Interaction entre un électron (sur deux états atomiques «0» et «1») et un champs photonique.

Emission:

$$\langle n+1,"0" | H_{ém} | n,"1" \rangle = \beta \cdot \sqrt{n+1} \cdot \langle n+1,"0" || n+1,"0" \rangle$$

$$\left| \langle n+1,"0" | H_{ém} | n,"1" \rangle \right|^2 = |\beta|^2 \cdot (n+1)$$

Emission stimulée — Emission spontanée

Absorption:

$$\langle n-1,"1" | H_{ab} | n,"0" \rangle = \beta \cdot \sqrt{n} \cdot \langle n-1,"1" || n-1,"1" \rangle$$

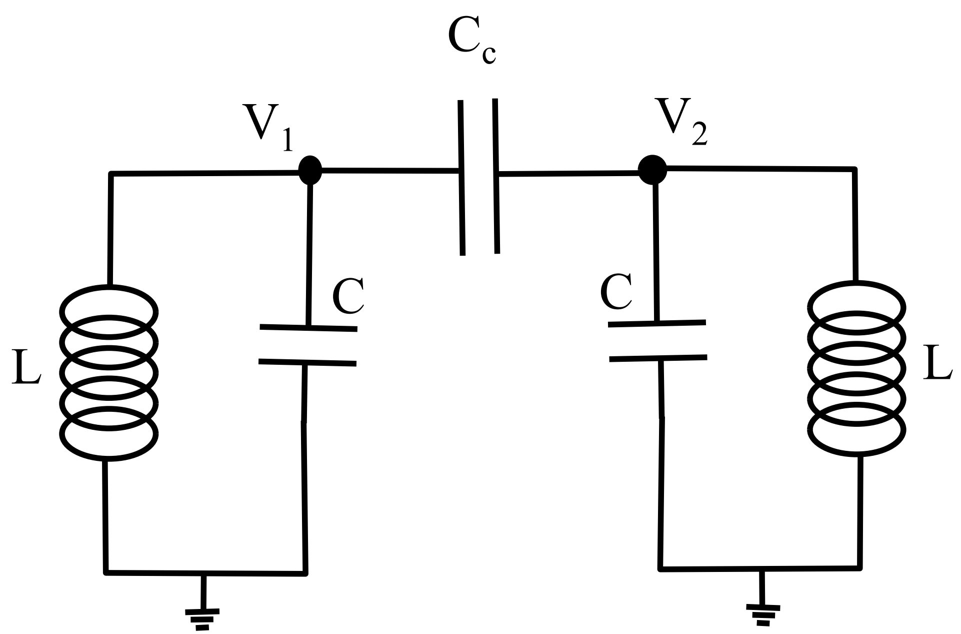
$$\left| \langle n-1,"1" | H_{ab} | n,"0" \rangle \right|^2 = |\beta|^2 \cdot n$$

Absorption

Exercice 12.1: Hamiltonien de couplage iSWAP

$$H_c \approx -T \cdot \left(a_{1+} a_{2-} + a_{1-} a_{2+} \right)$$

$$H_c \approx -T \cdot \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



Reprenons l'exercice 9.3:
Considérez seulement deux modes dans chaque résonateur (2 qubits)

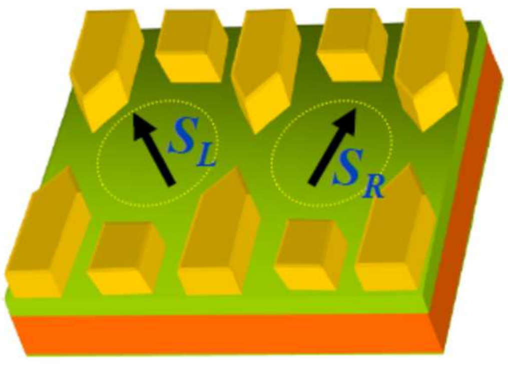
$$iSWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1) **Déterminez le propagateur U(t)**

2) **Déterminez le temps t_1 pour obtenir la fonction iSWAP**

Exercice12.2 : Hamiltonien de couplage $\sqrt{SWAP}$

Reprenons l'exercice 9.4



$$H = -T \cdot \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

- **Ecrivez l'Hamiltonien dans sa base de vecteurs propres**
- **Exprimez le propagateur dans la base des vecteurs propres**
- **Exprimez ce propagateur dans la base standard**
- **Quel temps t_1 permet d'obtenir la fonction $\sqrt{SWAP}$?**

$$\sqrt{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(1+i) & \frac{1}{2}(1-i) & 0 \\ 0 & \frac{1}{2}(1-i) & \frac{1}{2}(1+i) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$